Microlensing: Theory, Practice, Results, Future

Lecture 3

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Outline of Lectures

Lecture 1: Microlensing History and Theory

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- Background
- Motivation/Goals
- Early results
- Evolution of a field
- Basic microlensing theory

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Lecture 1: Microlensing History and Theory

Lecture 2: Beyond the Single lens

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Lecture 2: Beyond the Single lens

- Finite source star
- Limb Darkening
- Blending
- Parallax
- Xallarap

Outline of Lectures

Lecture 1: Microlensing History and Theory

Lecture 2: Beyond the Single lens

Lecture 3: Planetary Microlensing - I

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Lecture 1: Microlensing History and Theory

Lecture 2: Beyond the Single lens

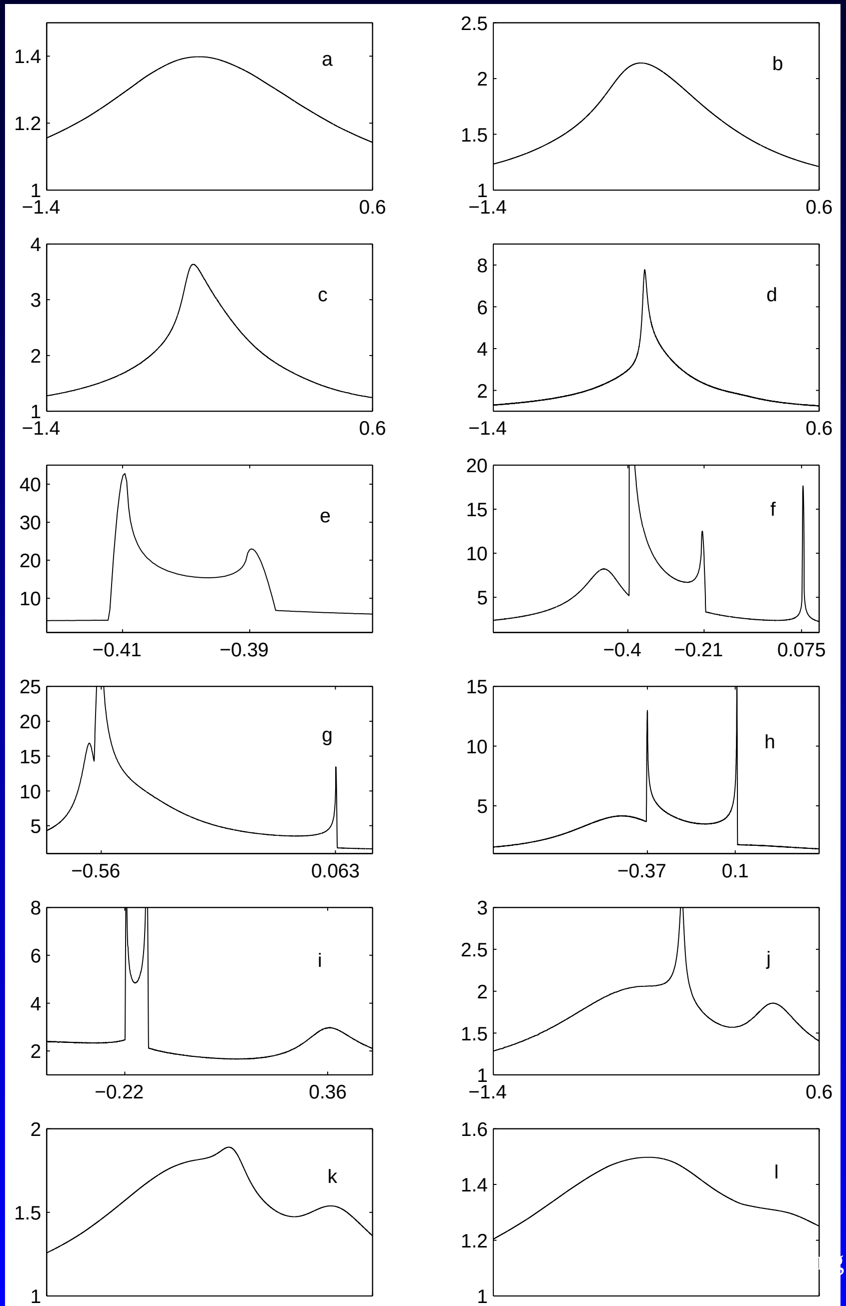
Lecture 3: Planetary Microlensing - I

- Binary lens microlensing
- Extreme mass ratio microlensing
- Theoretical tools of trade: caustics
- Planetary microlensing regiemes
- General rules

Binary lens microlensing

When the lens system is comprised of more than one object, strong departures from a smooth, symmetric single lens light curve is observed.

Binary lens microlensing



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In addition, of course, to the “single lens” parameters: $u_{\min}$, t_0 and t_E, and the flux parameters F_{u} and F_{l}.

Other effects such as finite source star size, source star limb-darkening, parallax and/or xallarap may also be important.

Lens Equation

The lens equation for n lens masses can be expressed as (Bozza 1999):

$$\mathbf{y} = \mathbf{x} - \frac{m_1 \mathbf{x}}{|\mathbf{x}|^2} - \sum_{i=2}^{n} \frac{m_i(\mathbf{x} - \mathbf{x}_i)}{|\mathbf{x} - \mathbf{x}_i|^2}$$

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The primary star, mass m_1, is placed at the origin of the lens plane and $\mathbf{x}_i$ is the position of the ith lens mass, m_i.

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where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ denote position in the lens and source planes respectively.

The locations of the other lens masses $\mathbf{x}_i$ are the projected positions onto the lens plane.

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where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ denote position in the lens and source planes respectively.
Provided the scale of the lens system is much smaller than the distances from the lens system to the observer and source, the projected positions of the lens masses onto the lens plane can be used instead of preserving the volume mass distribution.

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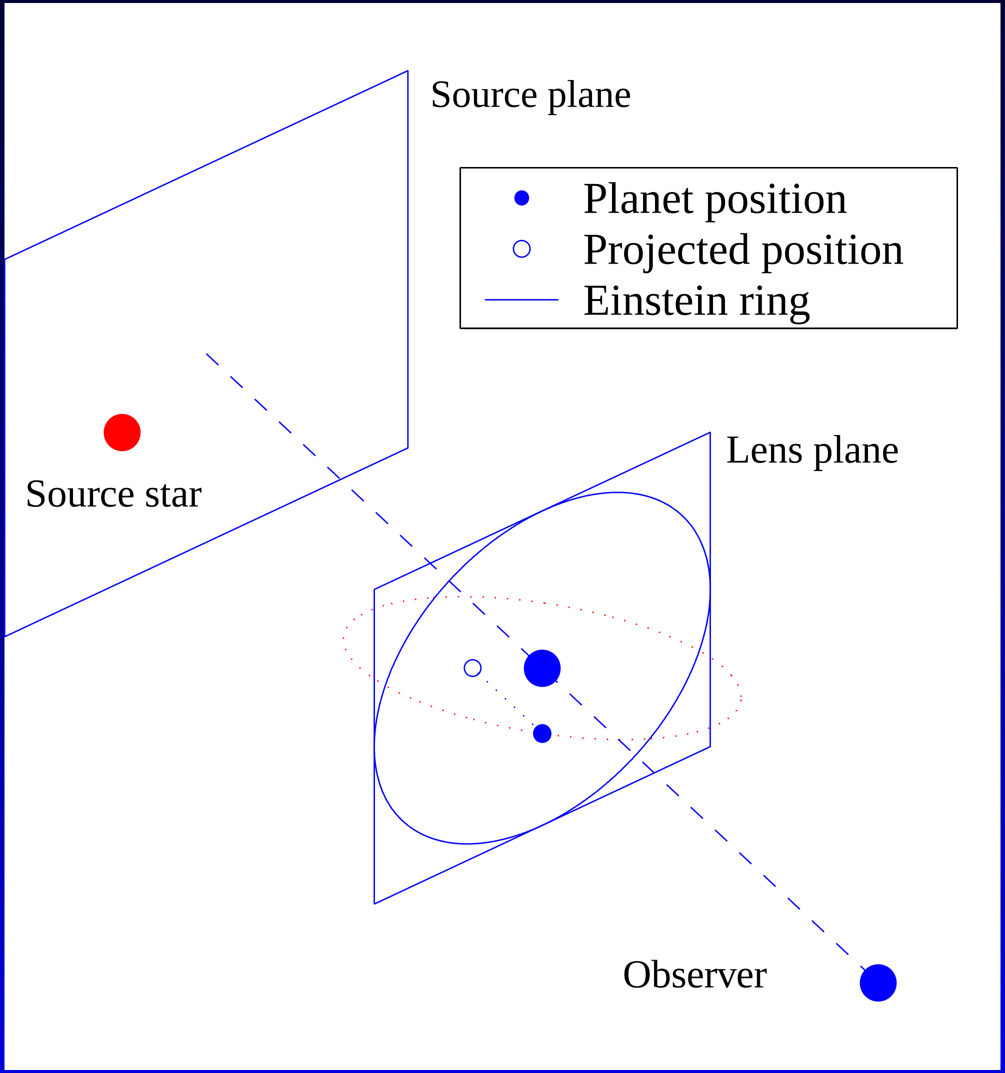
where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ denote position in the lens and source planes respectively.

Given a position in the source plane, $\mathbf{y}$, the values of $\mathbf{x}$ which satisfy Eq. 4 are the images in the lens plane of the source.

Recall that we want to solve for the images, as the ratio of image size to unlensed source size gives the amplification due to lensing (Lecture 1).

Lens Equation



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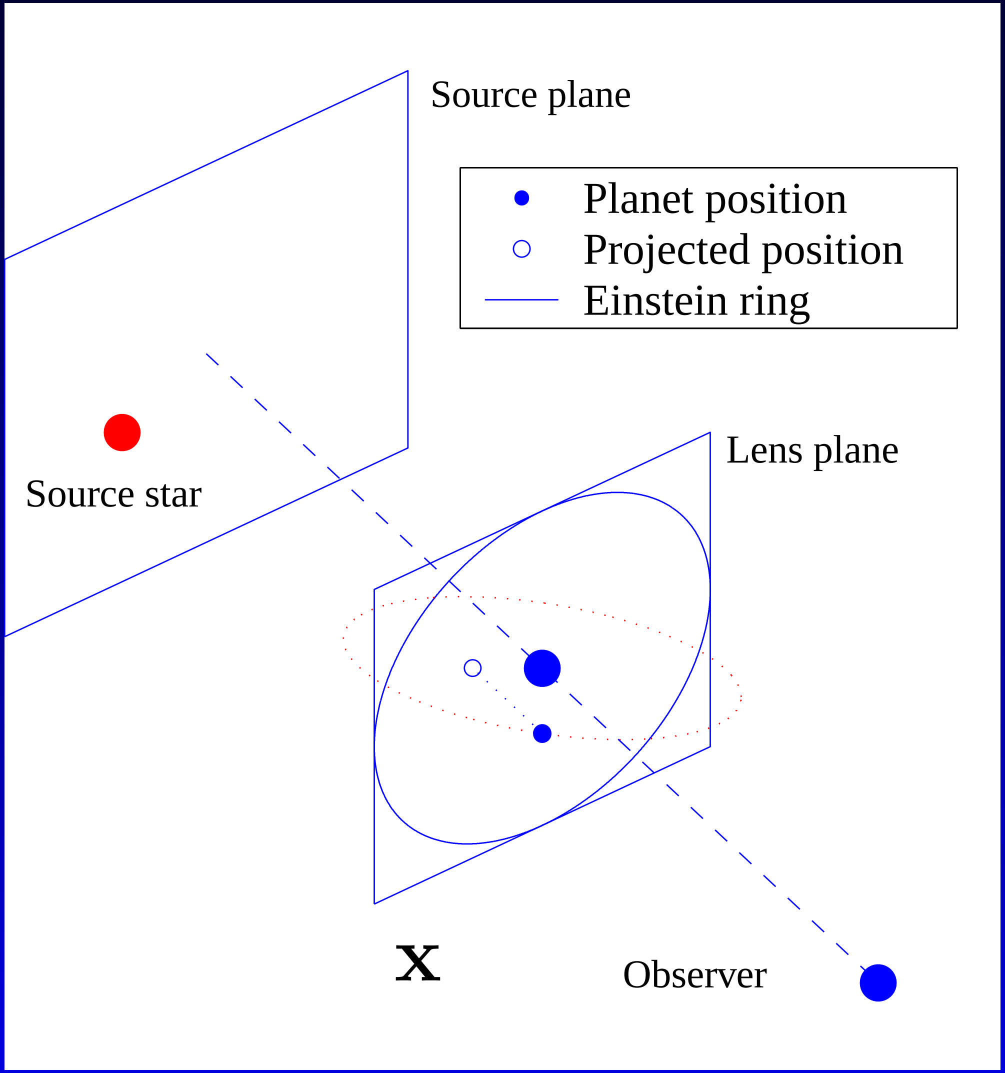

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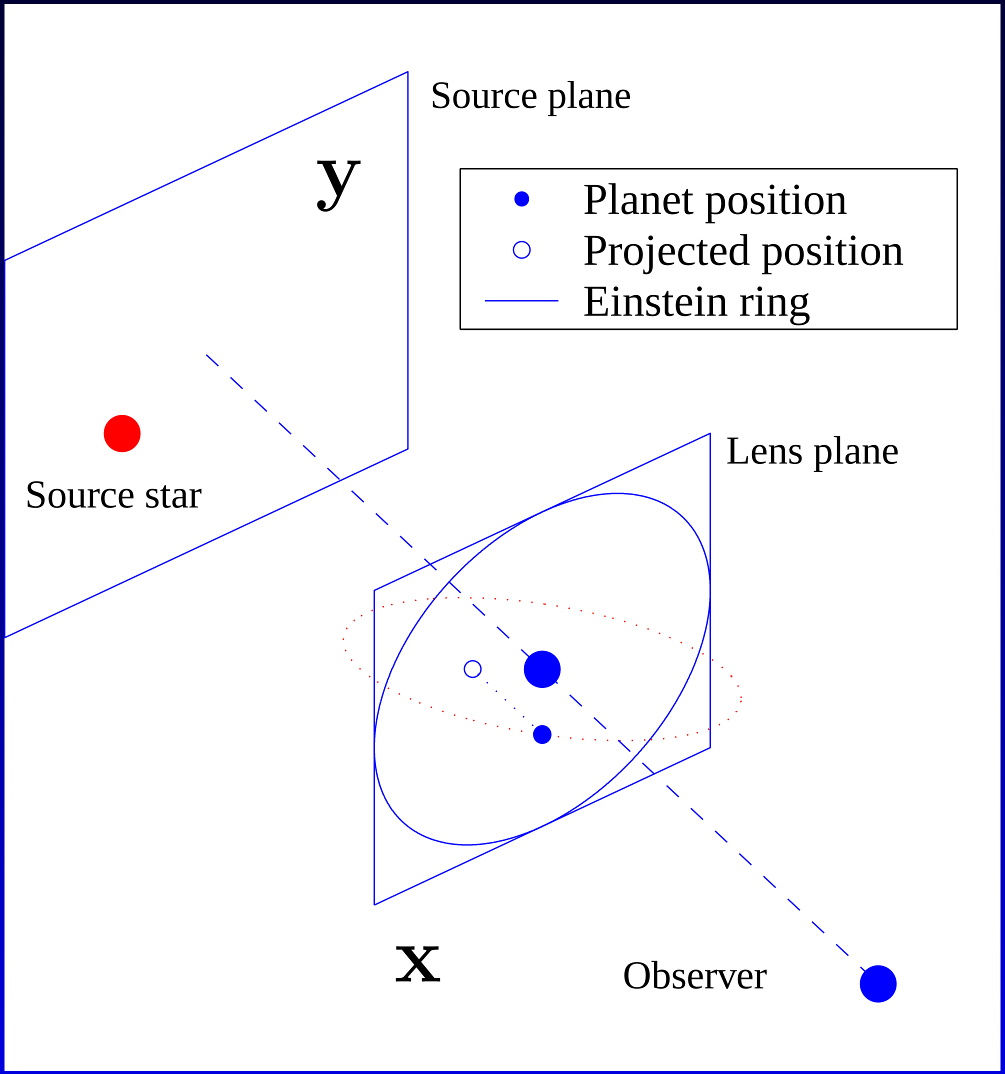

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- Inverting the lens equation gets progressively trickier for higher n.
- Inclusion of finite source star size becomes progressively harder.

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The non-linear effect of the lensing physics on microlensing lightcurves can be visualised.

Critical and Caustic curves

The critical curves are defined as the set of points in the lens plane for which the Jacobian of the lens mapping vanishes:

$$\mathbf{x}_{critical} = \{\mathbf{x} : \det(J) = 0\} \qquad J = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{pmatrix}$$

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The caustic curves are the set of points in the source plane to which the critical curves are mapped via the lens equation (Mao 1991).

$$\mathbf{y}_{caustic} = \{\mathbf{y} : \mathbf{y} = f(\mathbf{x}_{critical})\}$$

where $f(\mathbf{x})$ is the lens equation.

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Whilst not infinite, the nevertheless large and rapid peaks in the binary lens light curves correspond to the source star crossing one of these caustic lines.

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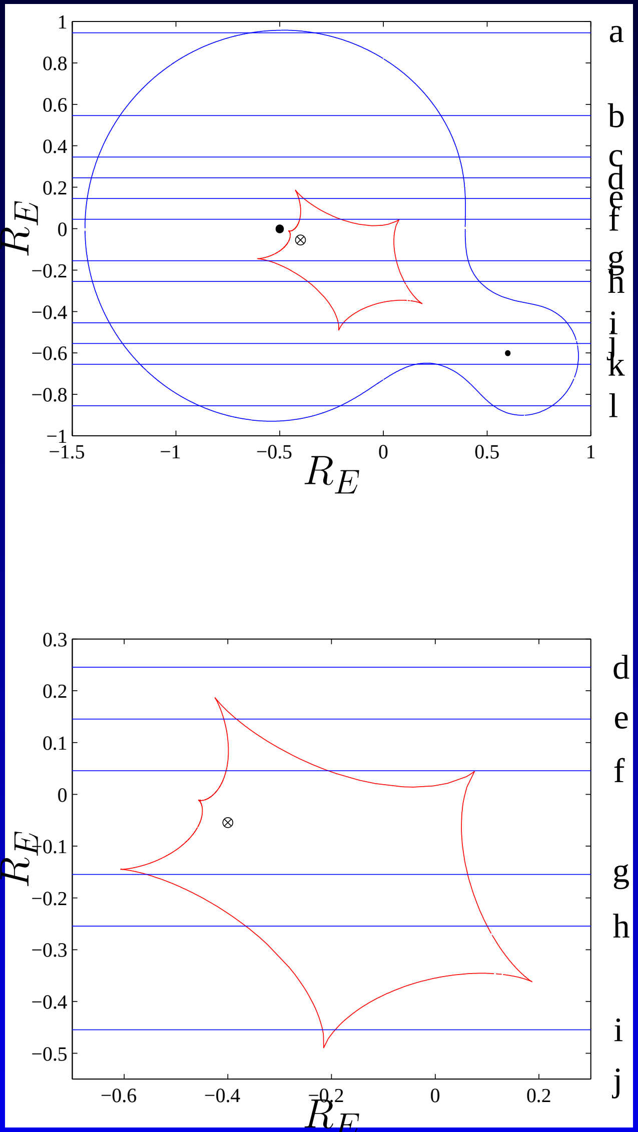
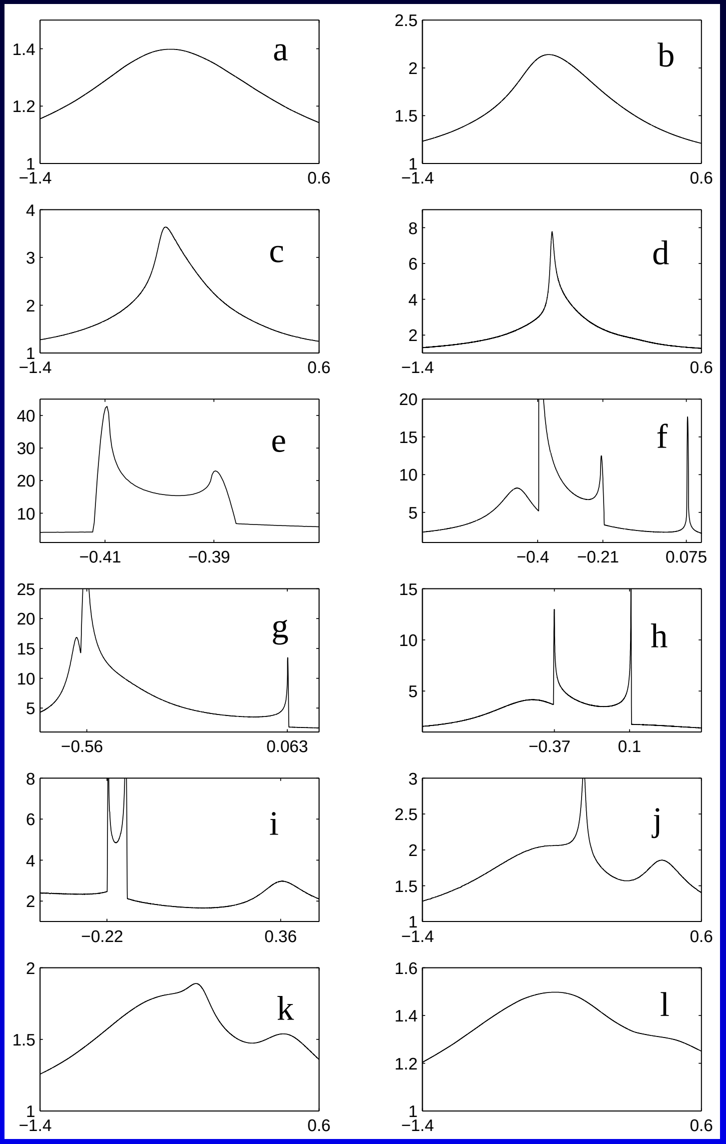
The amplification of the source star at this point is again, formally infinite, corresponding to to the situation where the source, lens and observer are collinear.

Other strong deviations are obvious in the light curves, particularly when the source star passes close to a caustic line cusp.

Critical and Caustic curves





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When the source intersects a caustic line, two images merge at a critical curve, therefore the total number of images changes by two as the source crosses a caustic (Chang 1979), producing the light curve spikes (Mao 1991, Dominik 1991).

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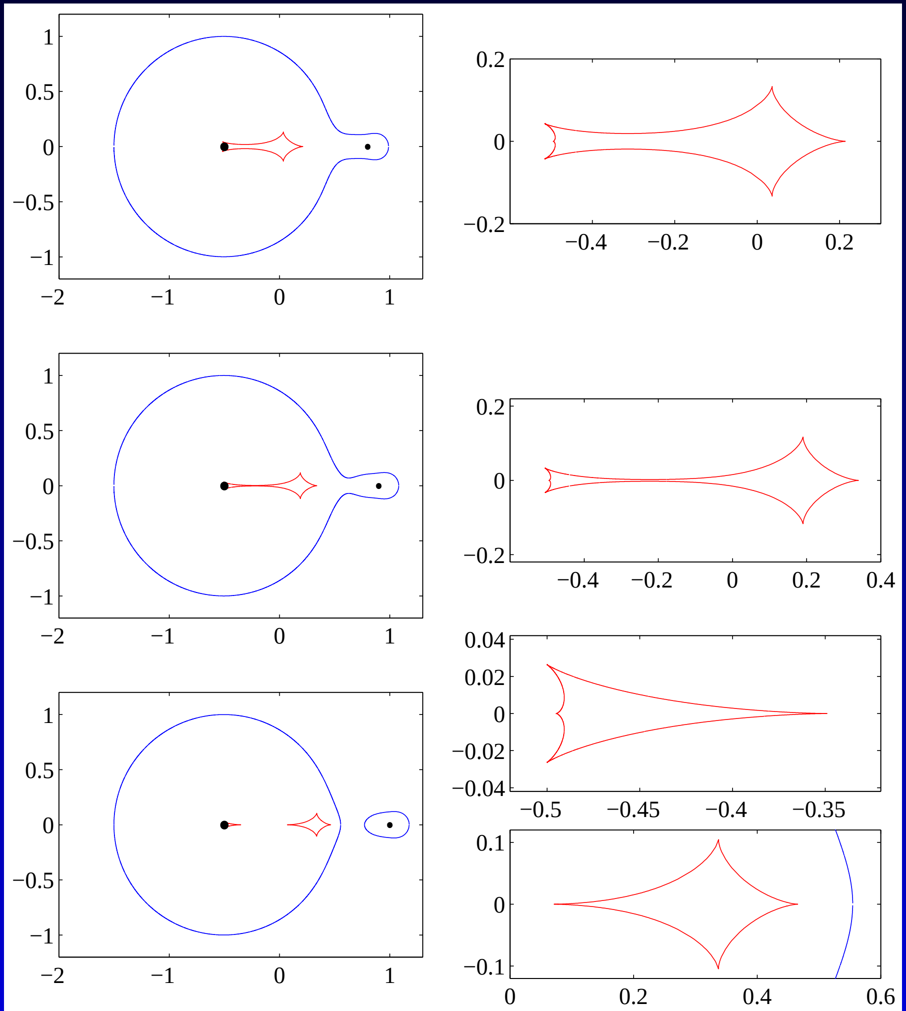
In the single lens case, when the source hits the caustic point, a full Einstein ring appears at the critical curve.

Critical and Caustic curves

The size of the caustic curves, relative to the Einstein ring, changes according to the mass ratio between the two lens components and their position in the lens plane.

Critical and Caustic curves



$$a_p > R_E$$

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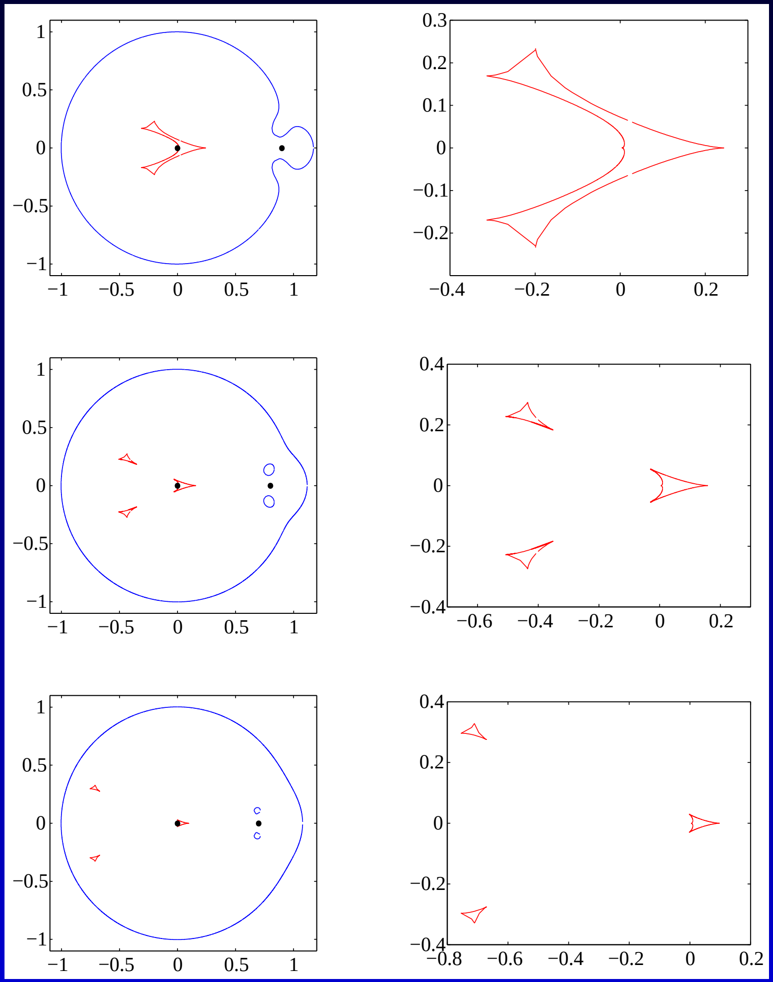
The next plot shows the critical and caustic curves for a planet which is positioned inside the Einstein ring of the primary mass, $a_p < R_E$.

Critical and Caustic curves



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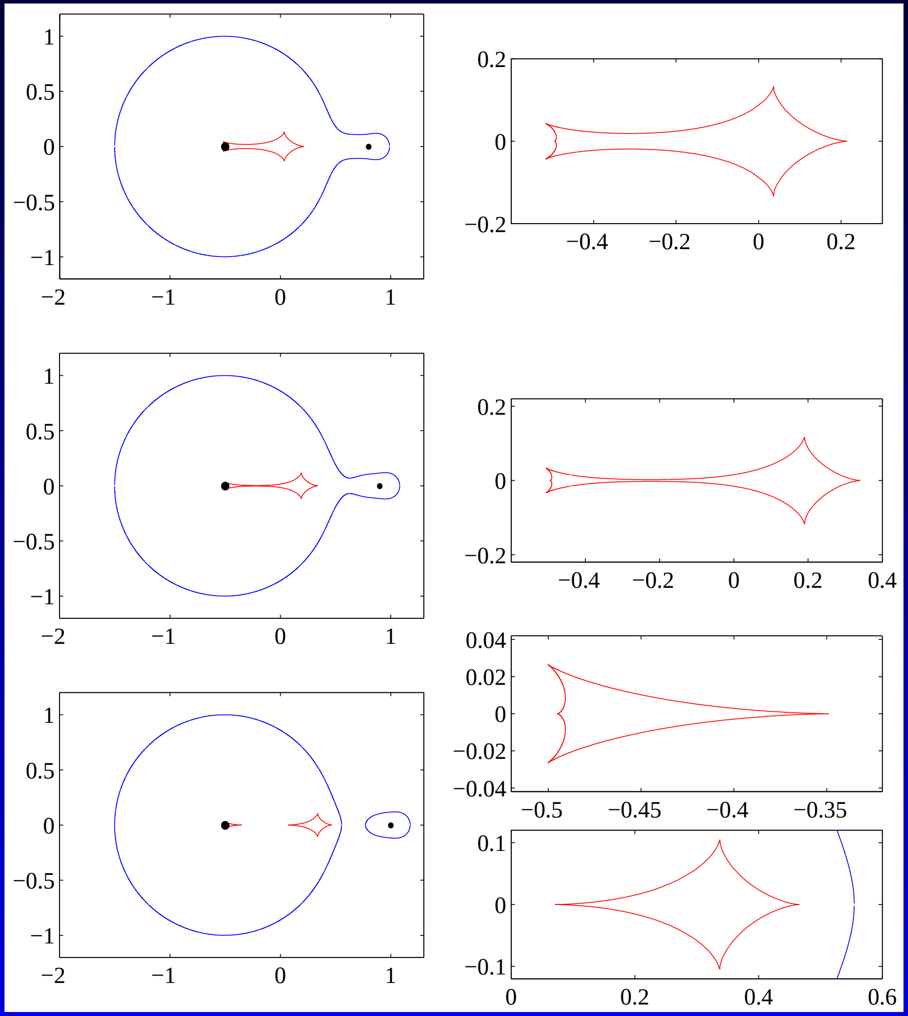

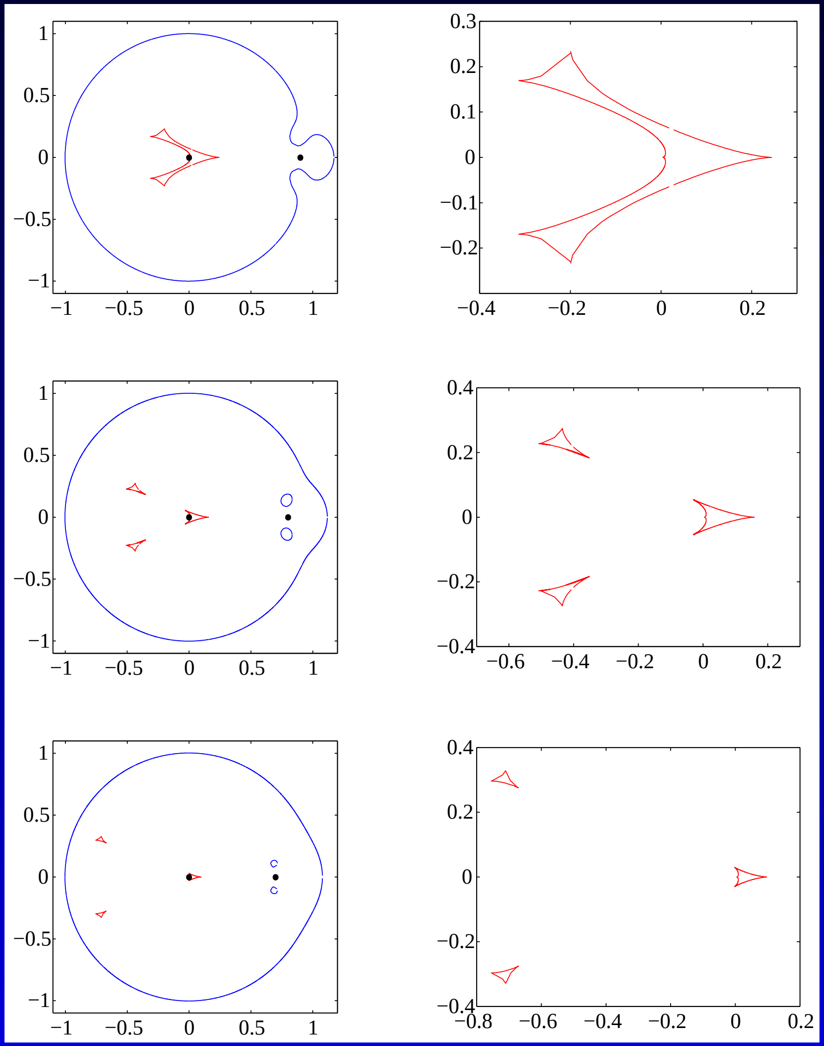

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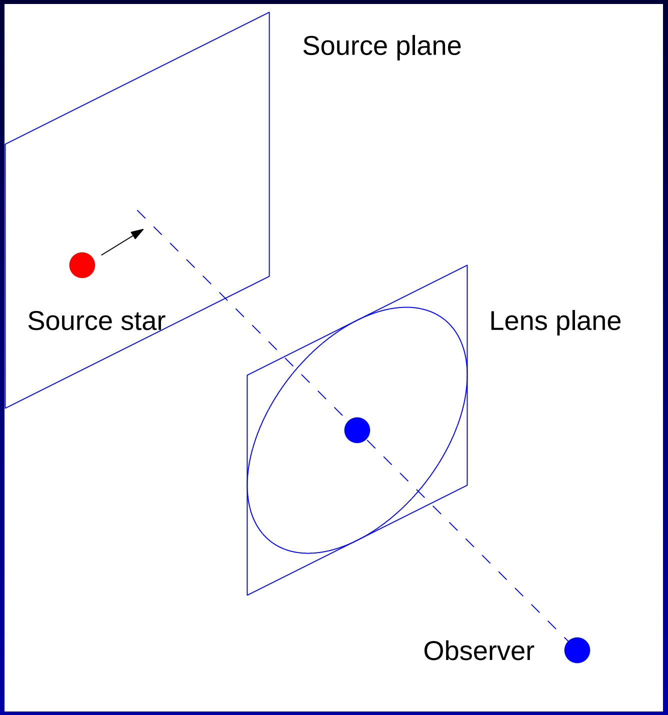
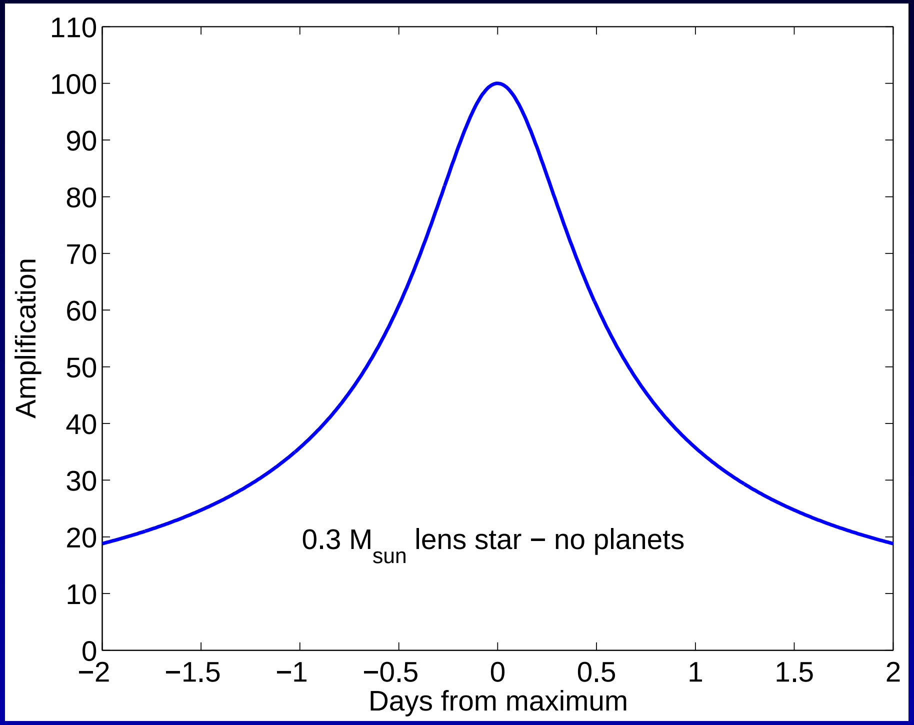
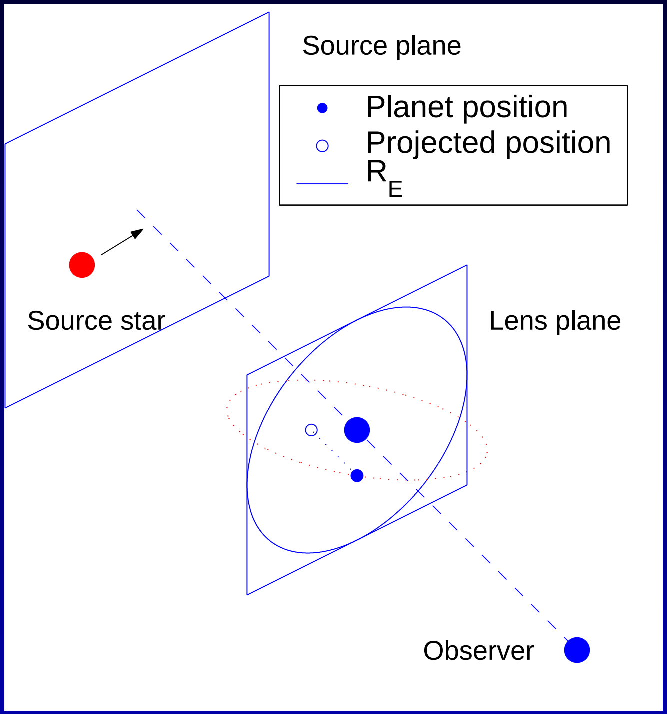
Planetary microlensing

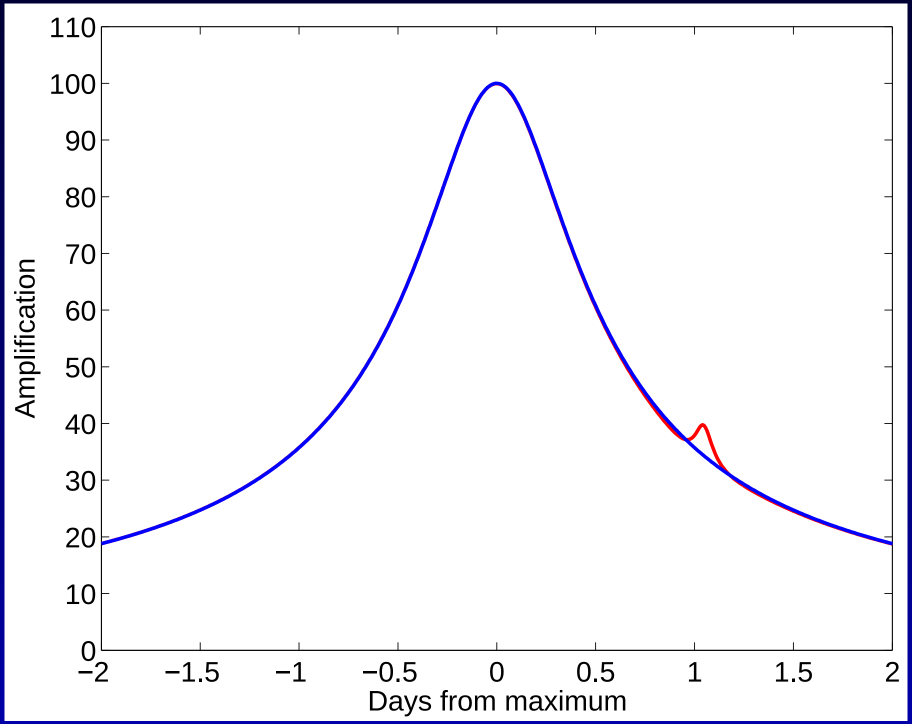




Relative motion of source, observer and lens gives time-dependent amplification of source.
t_E varies, typical timescale $t_E \simeq 20$ days.

Planetary microlensing





Presence of a planet orbiting the lens system can produce a perturbation in the light curve.

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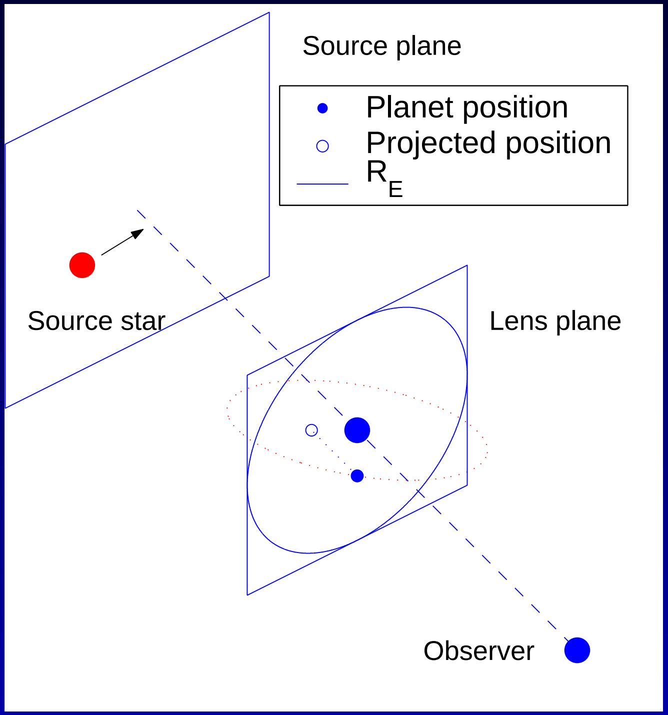

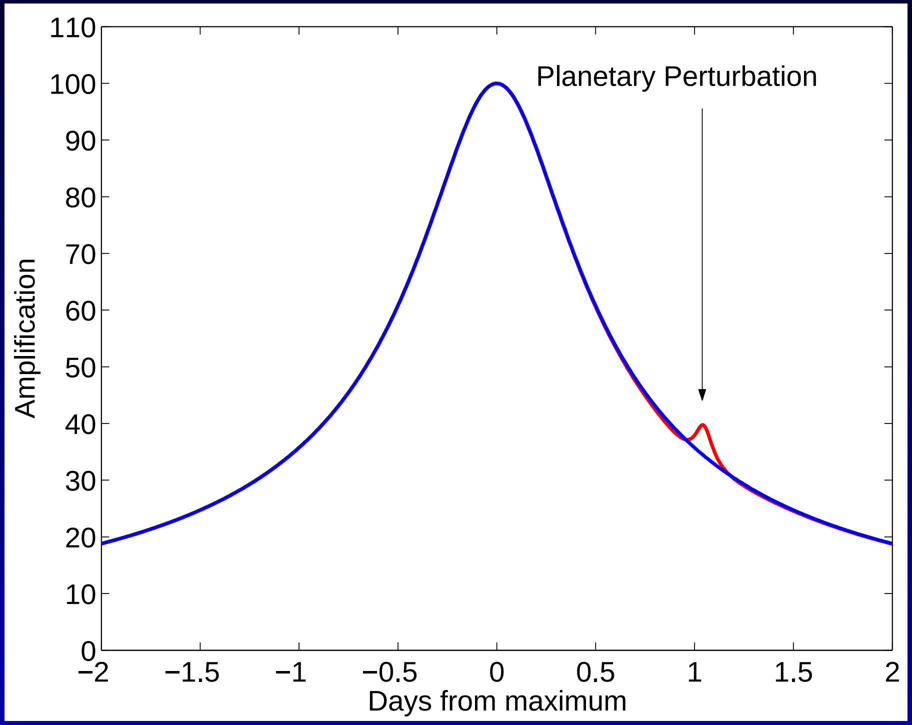

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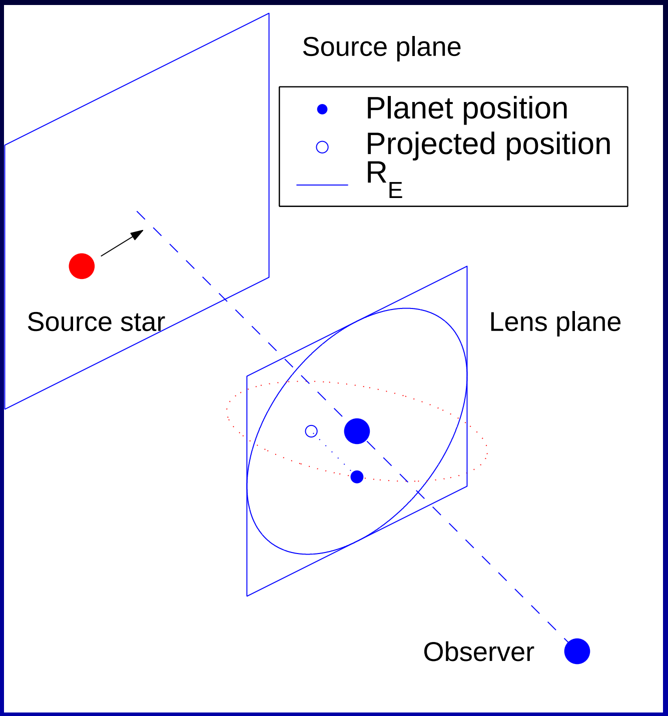

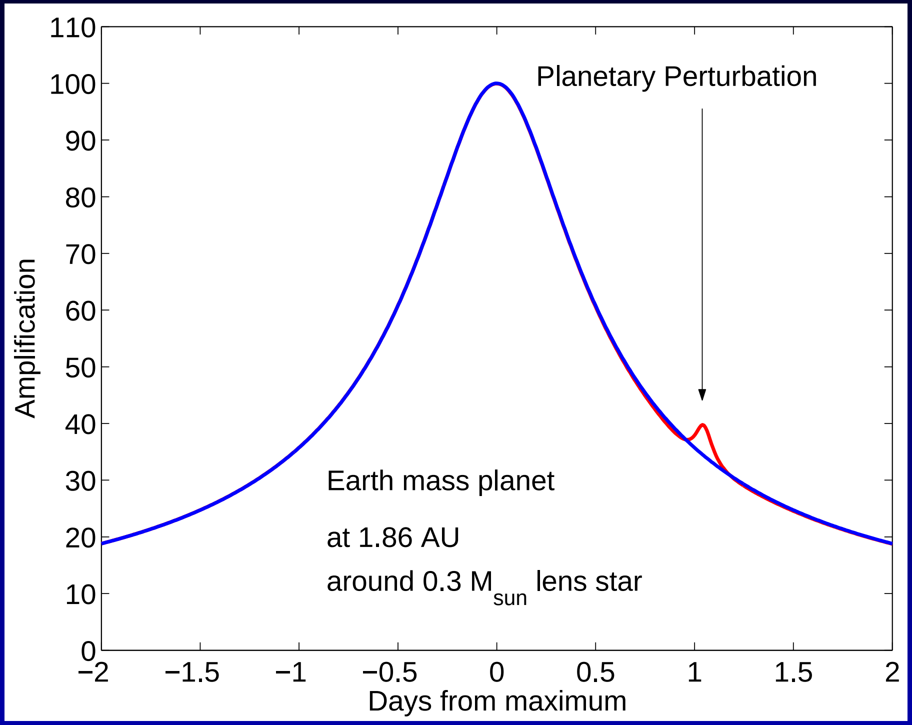

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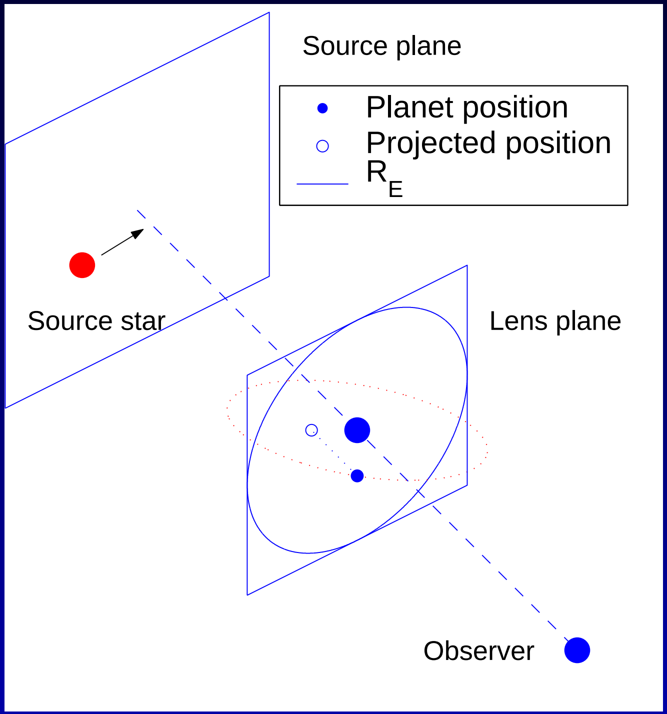
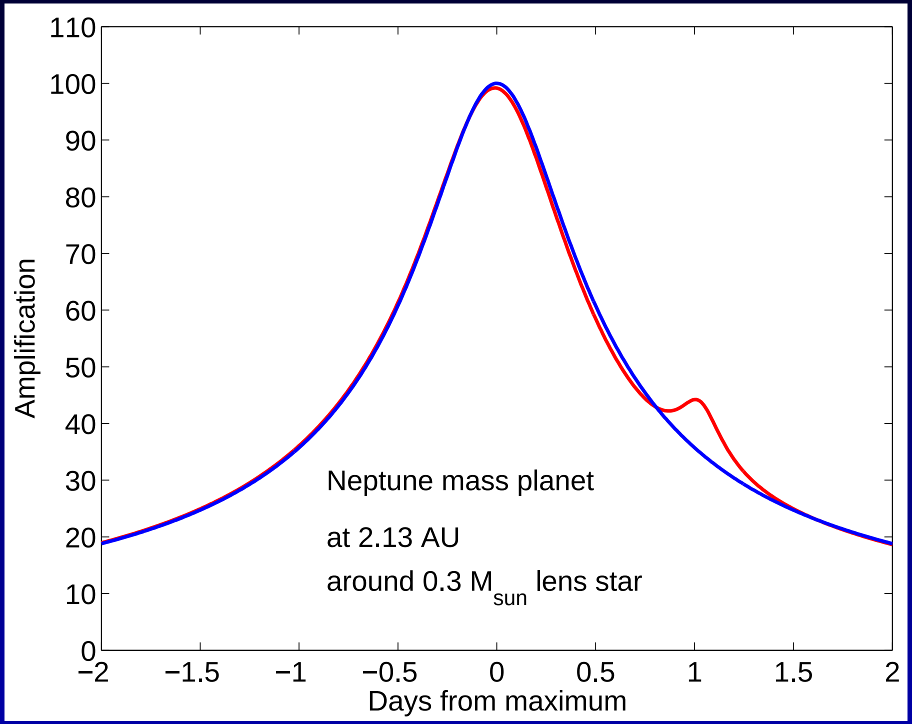
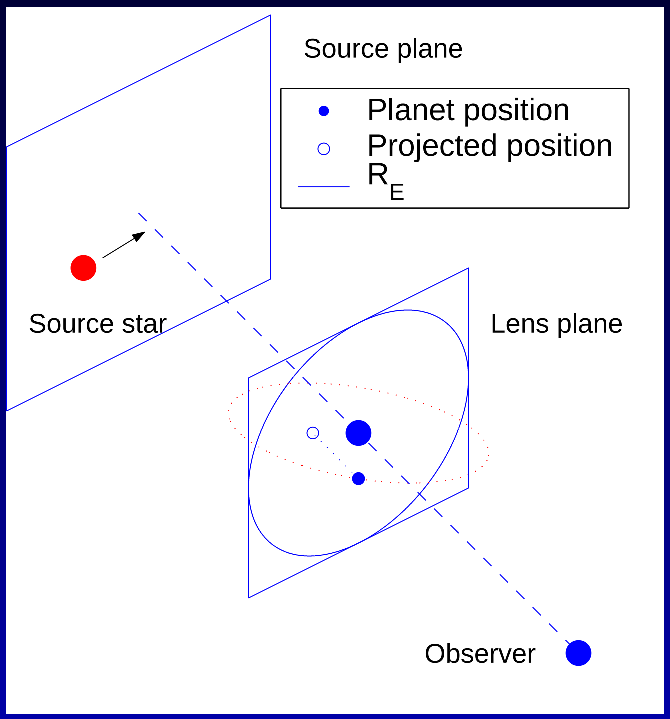
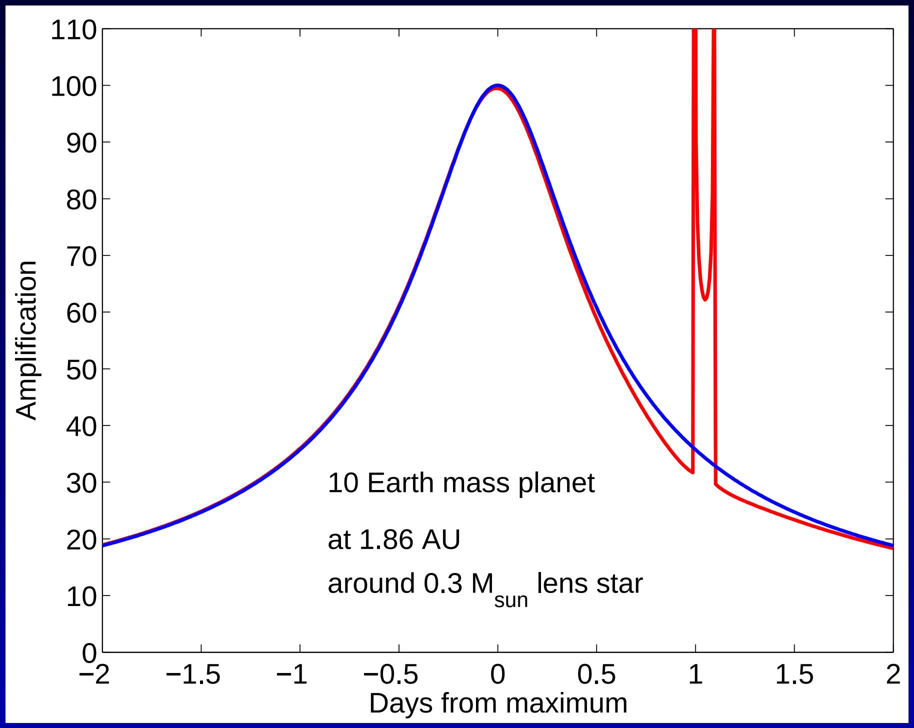




The features of the planetary perturbation depends on the mass of the planet relative to the mass of the lens star, its position angle, and orbit radius.

Planetary microlensing





These signals can be quite large, even for low-mass planets.

Planetary microlensing

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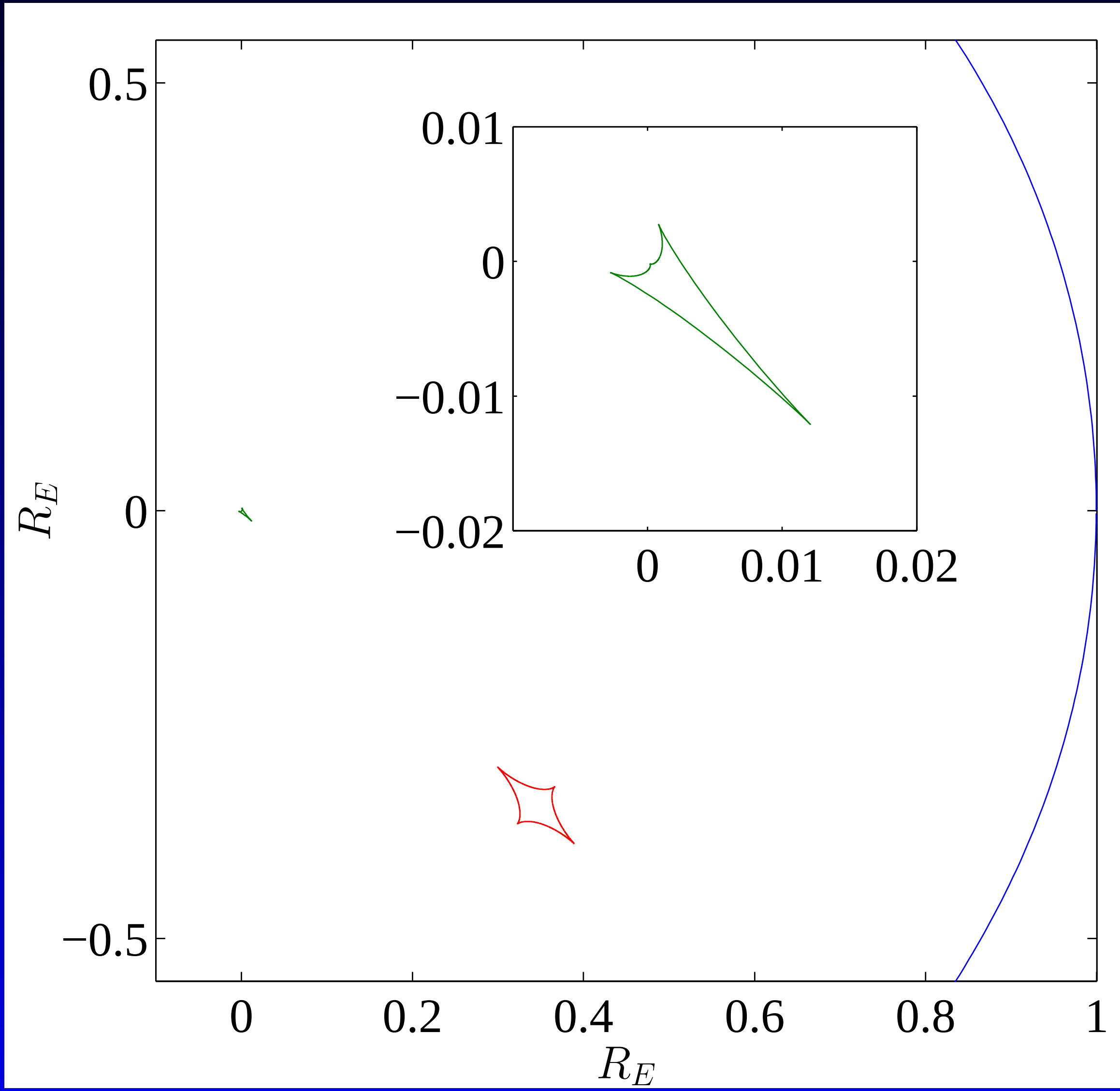
- Or the source star passes close to, or through both caustics.

Planetary microlensing



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The properties of these two types of planetary microlensing events have implications for how to conduct the search planets via microlensing.

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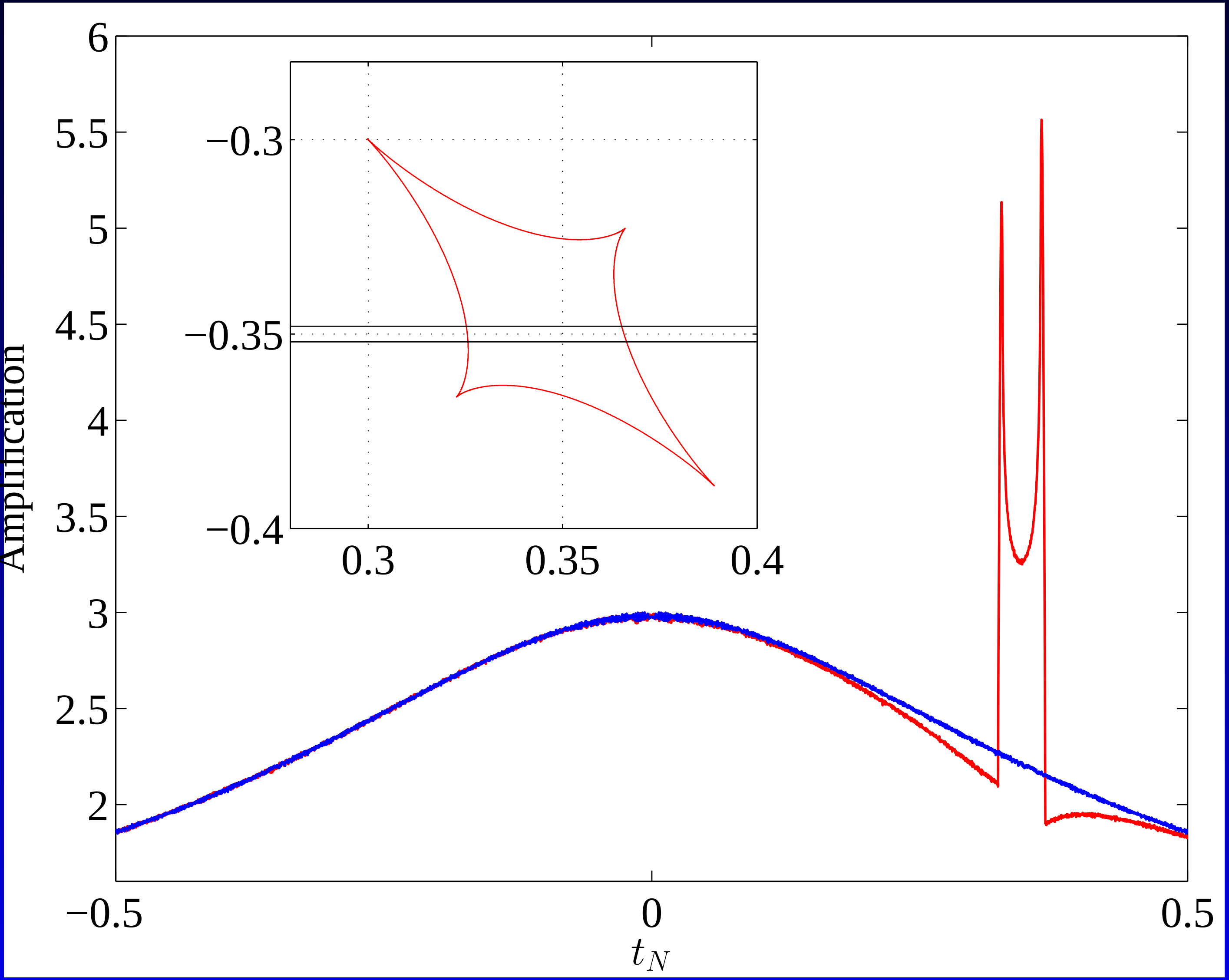

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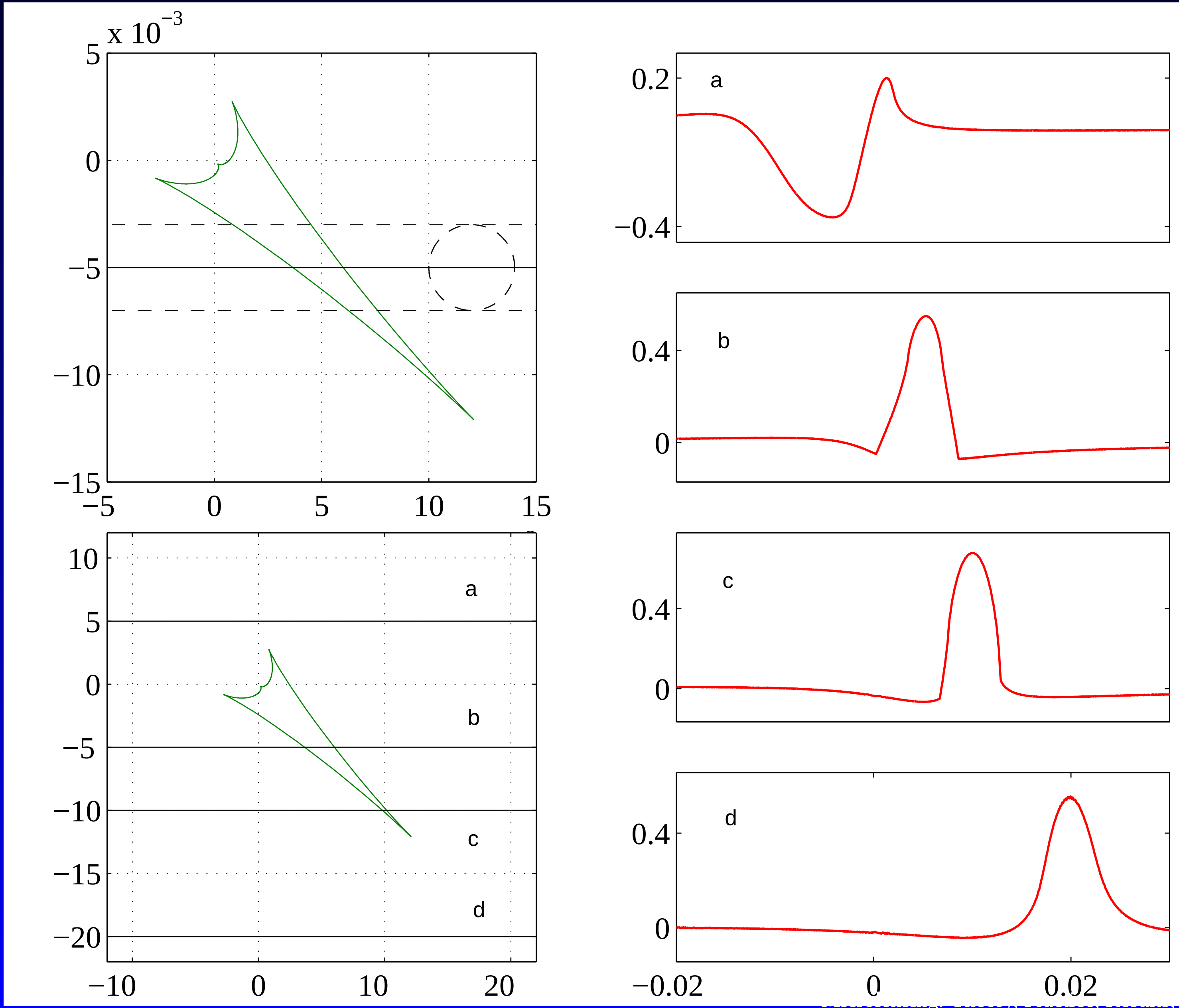

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The light curve deviations due to a source star passing close or through the central caustic are also varied, but due to the relatively small size of the central caustic, the deviations in the light curve will be most likely due to the source star passing close to a cusp of the central caustic curve. Source size effects are usually important.

Planetary microlensing

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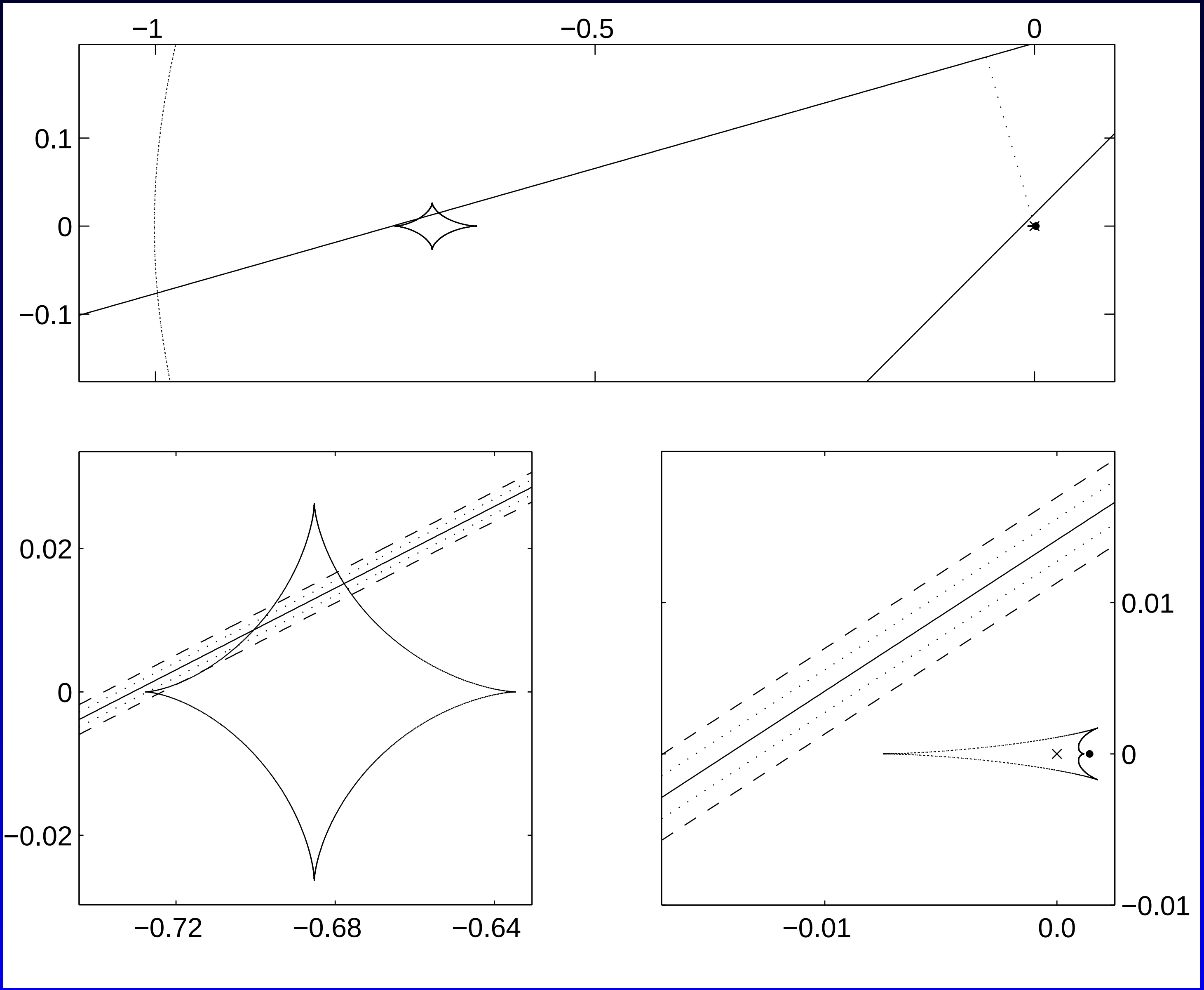
Compare this to planetary caustic crossing events which occur whenever the source happens to pass through a planetary caustic. The large excursions from the single lens light curve occur at essentially any time during the microlensing event, corresponding to where the planetary caustic lies in the source plane.

Planetary microlensing



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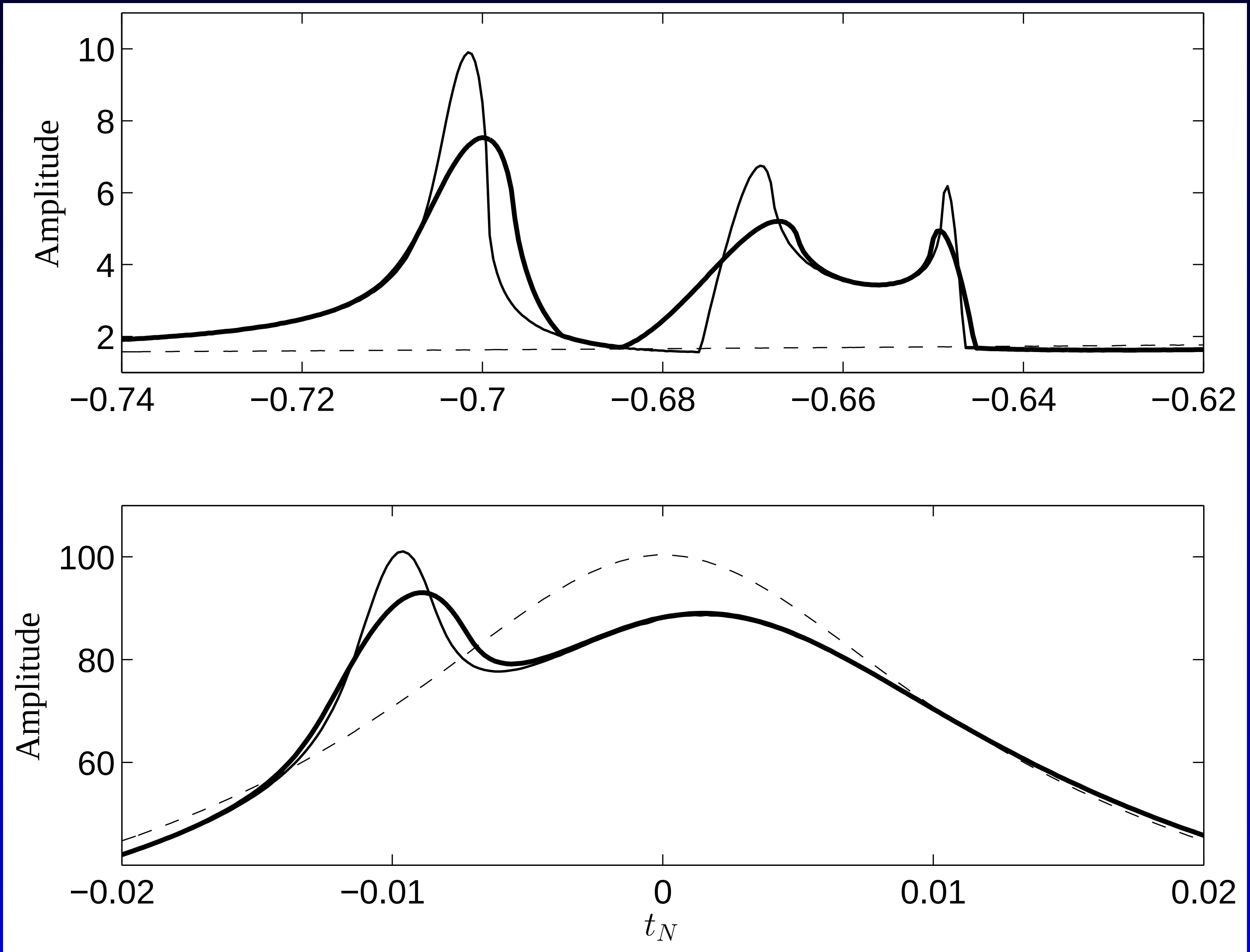

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In contrast, the strength of deviations due to planetary caustic approach or transit is relatively high.

Planetary microlensing

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This linear treatment is not applicable for caustic crossing events of any type.

Two modes

Technique	Advantages	Disadvantages
Planetary caustic crossing events	• Low amplification events are more common • Caustic crossing perturbations are large	• Requires moderate sampling over longer periods • Caustic crossing events are rare

Two modes

Technique	Advantages	Disadvantages
Peak perturbations in high amplification events	• Perturbations occur in well defined time interval • Planets are detected with higher efficiency • Sensitive to giant planets at almost any orbital radius	• Perturbations are relatively small • High amplification events are rare • Sensitive to terrestrial planets near the Einstein ring only

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In general however, microlensing is most efficient at detecting planets within an annulus around R_E. Park et al (2006) gives a lensing zone $0.6R_E \lesssim d \lesssim 1.6R_E$.

OGLE-2003-BLG-235/MOA-2003-BLG-53

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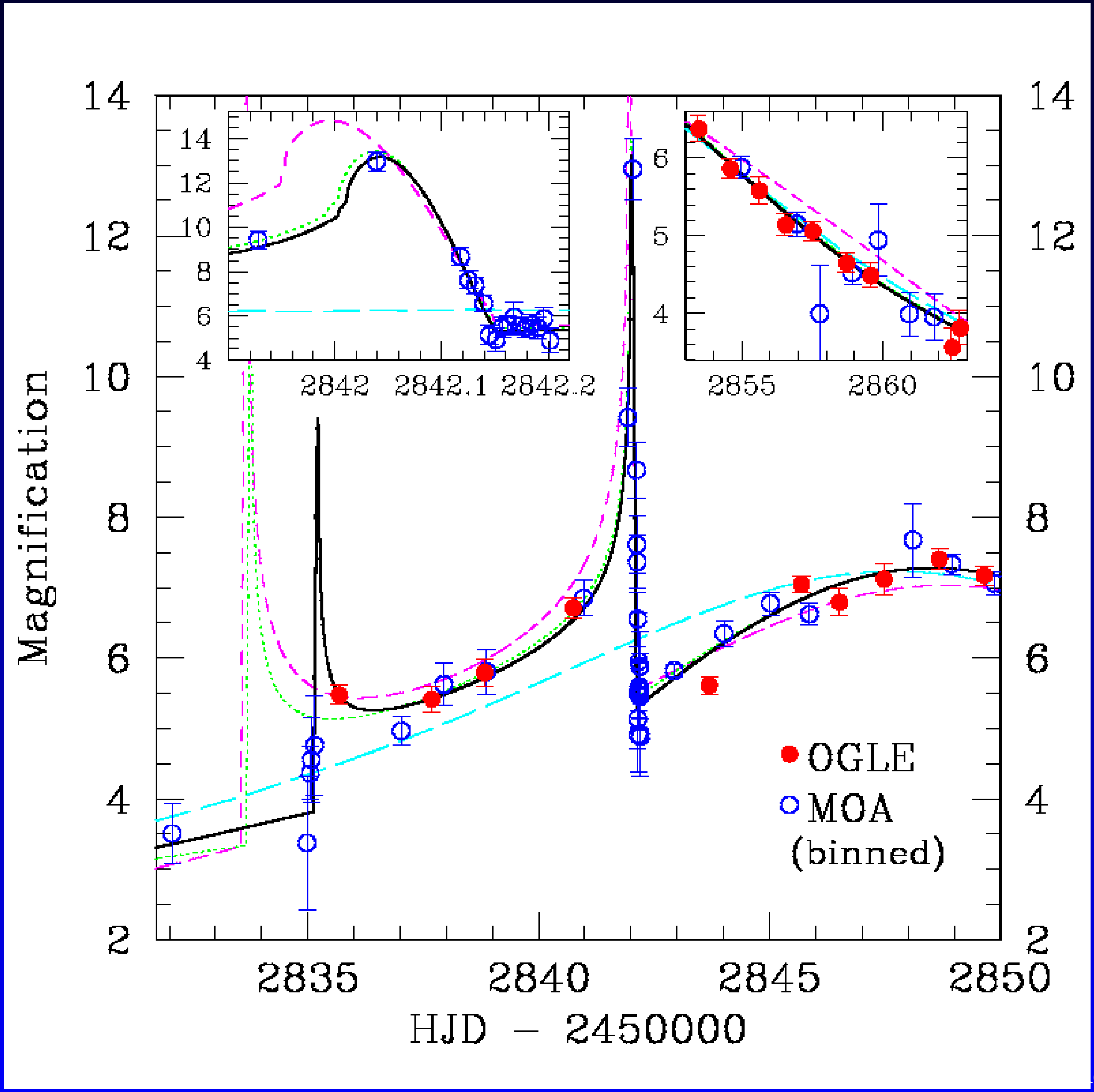
A clear caustic crossing signature is seen in the data. Several classes models were used to fit the data; an early caustic crossing model (green/dotted), a non-planetary model, i.e. $q > 0.03$ (magenta/dashed) and the best-fitting model (black/solid). The best-fitting single lens model is also shown (cyan/long dash).

Following figure reprinted from Bond et al (2004), courtesy D. Bennett. See also Bennett et al (2006).

OGLE-2003-BLG-235/MOA-2003-BLG-53



OGLE-2006-BLG-169

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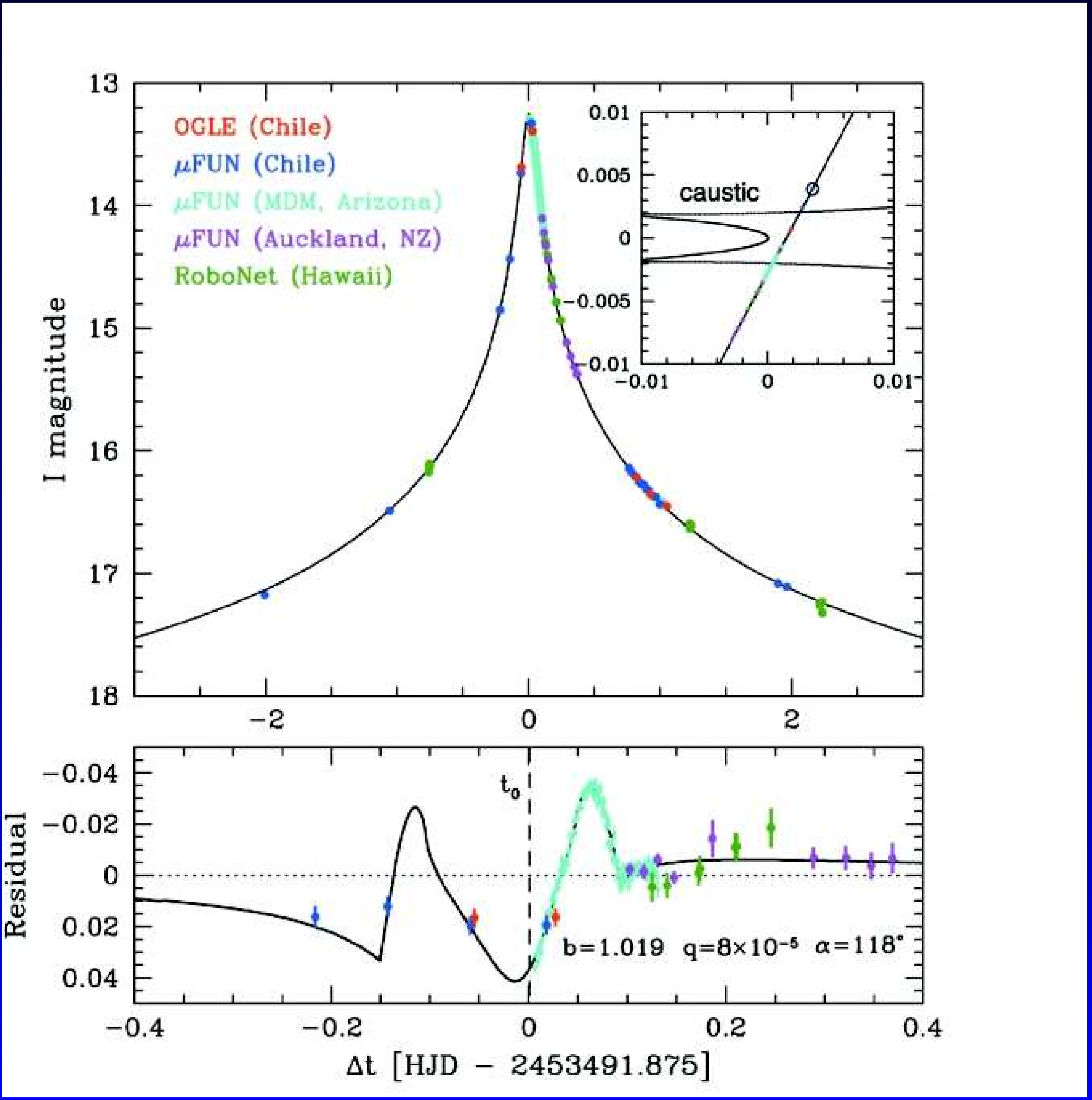

More later

These and other events are described in more detail in Rattenbury et al (2006).

and in later lectures.

Latest results will be presented as the proceedings of The Manchester Microlensing Conference 2008.