Weak Gravitational Lensing: A unique tool for probing Dark Matter and Dark Energy

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Who am I?

- 1996-2000 Masters in Physics from the University of Edinburgh, UK.
- 2000-2003 PhD from the University of Oxford, UK.
- 2003-2005 Postdoctoral Fellow at the Max-Planck Institute for Astronomy in Heidelberg, Germany.
- 2005-2008 CITA Fellow and Marie Curie Fellow at the University of British Columbia, Vancouver, Canada
- 2008.... Senior Fellow of the Institute for Astronomy at the University of Edinburgh, UK.

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Lecture 1: Weak lensing for Cosmology
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Lecture 1: Outline

• Basics of Cosmology: Dark Matter and Dark Energy
• Basics of Lensing: the lens equation
• Shear and Convergence
Basics of Cosmology

- Observations of the Cosmic Microwave Background, and distant Supernova 1a tell us that the Universe's expansion is accelerating.

- This implies that the Universe is currently dominated by a mysterious component called Dark Energy in addition to an invisible component called Dark Matter.

The big questions:
What is Dark Energy?
What is Dark Matter?
Zwicky discovered that the Coma cluster galaxies were moving much faster than expected.

There is some invisible matter that is causing the galaxies in the Coma cluster to stick together.

1933: Zwicky proposes the existence of “dark matter”.

The Coma Cluster
Spiral Galaxy: M31

\[ \frac{mv^2}{R} = \frac{GMm}{R^2} \]

Distance: R
Velocity: V

Extra matter which we can’t see!

Centripetal = Force due to force to gravity

\[ m \frac{v^2}{R} = \frac{GMm}{R^2} \]

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Lecture 1: Weak lensing for Cosmology
Evidence that a massive halo of Dark Matter surrounds all galaxies and clusters of galaxies.
The Cosmic Web of dark matter

The Millennium Simulation
(MPA Garching)
What would cause the Universe to accelerate?

The expansion of the Universe, can be derived from Einstein’s gravitational field equations, and is given by the Friedmann equation;

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) \]

If the Universe is accelerating there must be a component that has negative pressure.
What has negative pressure?

Think of a spring under tension. This tension is similar to a ‘negative pressure’ that can counteract gravity.

- Various types of dark energy have been proposed including:
  - a cosmic field associated with inflation
  - a different, low-energy field dubbed "quintessence"
  - the cosmological constant or vacuum energy of empty space.

\[
\rho_\Lambda \sim 10^{-26} \text{kg m}^{-3} \quad \rho_{\text{vac}} \sim 10^{95} \text{kg m}^{-3}
\]

- Modification to Einstein model of gravity
What has negative pressure?

In short: we have no idea what is causing the acceleration of the Universe, but we really want to find out!
Science Fiction or Science Fact?

• Why did dark energy become important around the time that we were here to think about it
  • A cosmic coincidence?
• Why is the majority of matter in the Universe weakly interacting and what is it?
  • Axioms, SUSYs?
• Is the answer to both these questions simply that our understanding of gravity is currently very poor?

Weak Gravitational Lensing is one of the most promising techniques available to try and answer these questions.
The Basics of Gravitational Lensing

- Lensing provides a direct detection and measurement of mass.
- It is the only way that we can observe dark matter on all scales.

$$\alpha = \frac{4GM}{c^2 r}$$
In the absence of a lens, the observer would see the source at position $\beta$. Instead, the lensing causes the observer to see the source image at position $\theta$. As all angles, in typical lensing situations, are very small,

$$\theta D_s = \beta D_s + \hat{\alpha} D_{ds}. \quad (1)$$

Defining the reduced deflection angle $\alpha = \hat{\alpha} D_{ds}/D_s$, the lens equation is given by

$$\beta = \theta - \alpha. \quad (2)$$
Strong lensing: Einstein Ring

From Einstein's theory of general relativity we know that

$$\hat{\alpha} = \frac{4GM}{c^2 \xi}. \quad (3)$$

In the case of an Einstein ring, $\beta = 0$ and the observed Einstein ring radius is equal to the reduced deflection angle $\theta_E = \alpha$. This allows one to immediately calculate the mass of the lens, using $\xi = D_d \theta$,

$$\theta^2_E = \frac{4GM_{<\theta_E}}{c^2} \frac{D_{ds}}{D_s D_d}. \quad (4)$$

In this case, the lens is at $z = 0.168$, the source is at $z = 1.55$ and the Einstein radius $\theta_E = 1.25''$. (Aragon-Salamanca & STAGES 2008)
The critical surface mass density $\Sigma_{\text{crit}}$

The surface mass density of the lens plane is defined as

$$\Sigma(\xi) \equiv \int dz \rho(\xi_1, \xi_2, z),$$

(5)

where $\rho$ is the density of the lens.

As an example, we will consider a circularly symmetric lens of constant surface mass density $\Sigma$. The mass contained in a radius $r$ is then given by $M = \Sigma \pi r^2$.

We can use the deflection angle equation to define a critical surface mass density $\Sigma_{\text{crit}}$ such that when $\Sigma = \Sigma_{\text{crit}}$ we see an Einstein ring.

$$\Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}}.$$  

(6)
Convergence $\kappa$

In practice, lenses are more complex but we will find $\Sigma_{\text{crit}}$ useful to define the dimensionless surface mass density or convergence $\kappa$

$$\kappa(\theta) = \frac{\Sigma(D_d \theta)}{\Sigma_{\text{cr}}}.$$  \hspace{1cm} (7)

A mass distribution which has $\kappa \geq 1$, will produce multiple strong lensing images for some source positions. $\kappa$ therefore distinguishes between the strong lensing regime ($\kappa \geq 1$) and weak lensing regime ($\kappa \ll 1$).

Hubble space telescope image of strong gravitational lensing by the galaxy cluster 0024+1654 (NASA HST archive).
The deflection angle for a mass distribution

In gravitational lensing theory, we assume the field is weak $\hat{\alpha} \ll 1$, and approximate the deflection produced by the total mass distribution as the sum of deflection angles from a series of point masses.

- Divide the mass distribution into cells of volume $dV$.
- Each cell then has a mass $dm = \rho(r)dV$.
- Consider a light ray propagating along the $z$ axis with position $(\xi, z)$ near a mass element $dm$ with position $(\xi', z')$.
- Use the Born approximation: near the deflecting mass approximate the light ray as a straight line with an impact vector $(\xi - \xi')$.
- Calculate the total deflection angle by summing over all mass.
The deflection angle for a mass distribution

\[
\hat{\alpha}(\xi) = \frac{4G}{c^2} \sum dm(\xi'_1, \xi'_2, z') \frac{\xi - \xi'}{|\xi - \xi'|^2}
\]

\[
= \frac{4G}{c^2} \int d^2\xi' \int dz' \rho(\xi'_1, \xi'_2, z') \frac{\xi - \xi'}{|\xi - \xi'|^2}.
\]

Defining the surface mass density of the lens plane

\[
\Sigma(\xi) \equiv \int dz \rho(\xi_1, \xi_2, z),
\]

we find the two dimensional vector of the deflection angle

\[
\hat{\alpha}(\xi) = \frac{4G}{c^2} \int d^2\xi'\Sigma(\xi') \frac{\xi - \xi'}{|\xi - \xi'|^2}.
\]
The deflection angle $\alpha$ is caused by a deflection potential, called the lensing potential $\psi$;

$$\alpha(\theta) = \nabla_\theta \psi(\theta). \quad (11)$$

Re-writing the deflection angle $\alpha$ in terms of the convergence $\kappa$

$$\alpha(\theta) = \frac{1}{\pi} \int d^2 \theta' \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2}. \quad (12)$$

and using some mathematical identities, we arrive at the simple relationship between convergence $\kappa$ and the lensing potential $\phi$

$$\kappa(\theta) = \frac{1}{2} \nabla^2_\theta \psi(\theta) = \frac{1}{2} (\psi,_{11} + \psi,_{22}), \quad (13)$$
The lensing Jacobian

The lensing Jacobian is a very useful way to describe the mapping between the source ($\beta$) and lens ($\theta$) plane.

\[
A_{ij} = \frac{\delta \beta_i}{\delta \theta_j} = \frac{\delta}{\delta \theta_j} [\theta_i - \alpha_i(\theta)]
\]

\[
= \delta_{ij} - \frac{\delta \alpha_i(\theta)}{\delta \theta_j}
\]

\[
= \delta_{ij} - \frac{\delta^2 \psi(\theta)}{\delta \theta_i \delta \theta_j}
\]

(14)

\[
A = \begin{pmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{pmatrix}
\]

(15)

where we have used equations 11 and 13 and introduced the complex shear of $\gamma$

\[
\gamma = \gamma_1 + i\gamma_2
\]

(16)
The complex shear $\gamma$ is not a vector, but a complex number of magnitude $\gamma$ and orientation $\phi$, where the real and complex components are related to the lensing potential:

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) = \gamma \cos(2\phi),$$
$$\gamma_2 = \psi_{,12} = \psi_{,21} = \gamma \sin(2\phi).$$ (17)

It is useful to define the reduced shear $g = \gamma/(1 - \kappa)$ such that

$$A = (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix},$$ (18)

This shows that convergence $\kappa$ only effects the size of the image and its magnification. The shear is responsible for distorting the shape of images.
Magnification

- Lensing conserves surface brightness. There is no process that emits or absorbs photons so as the light bundles evolve their density doesn’t change (Liouville’s theorem).
- If lensing increases the area of the image we therefore see magnification.
- But how can the lensed image be brighter than an unlensed source image?
- When the source is lensed we receive more photons than we would have detected in the absence of lensing.
- In addition to magnification we see distortion as light bundles are deflected differently.
We want to use lensing to measure mass.

We defined a quantity that measured the surface mass density (called the convergence $\kappa$).

We found that we cannot directly measure convergence.

But lensing also distorts or ‘shears’ galaxies $\gamma$.

We can relate the shear that we can measure to the convergence that we want to measure through the lensing potential $\psi$.

So, how do we measure shear? We have 4 days to learn!
The Abell Catalogue contains >4000 galaxy clusters detected by eye using the Palomar Optical Sky Survey in the 1950s.
A mass map of Abell 226/228: the goal of the workshop

- This data is part of the COMBO-17 survey (PI Wolf & Meisenheimer) and imaged in 5 broad bands and 12 narrow bands (good for photo-zs)
- Data reduced from the ESO archive by Thomas Erben.
- This image is BVR colour-composite
- Rule no1 when using archive data: CONTACT THE PI!
- COMBO-17 team are happy for us to use this data for teaching purposes. If you wish to continue using this data set for science you must inform them!

A228: \( z = 0.128 \)
Potential science with this data includes...

- With a shear catalogue and a photo-z catalogue (see Erben and Mobasher course for the latter) you could

- Investigate the shear-ratio test (constrain cosmology)

- Look at galaxy properties as a function of their dark matter environment

- Measure the dark matter halo profile of the galaxy clusters and the individual galaxy cluster members

Taylor et al 2003
A final word of motivation

- From the Dark Energy Task Force (Albrecht et al)
  - If systematic errors can be controlled, weak lensing is ‘likely to be the most powerful individual ... technique, and also the most powerful component in a multi-technique program’ for studying Dark Energy.

- From the ESA-ESO Working Group on Cosmology (Peacock et al)
  - Large scale weak lensing surveys with photometric redshifts have the best formal accuracy. However, all techniques may be subject to unanticipated systematic limits.

- Accurately measuring a weak lensing signal is a non-trivial task.

- This course will teach you one of the most important parts of a weak lensing analysis: how to extract an accurate and unbiased signal from real data.