Cosmology with non-minimal coupled gravity: inflation and perturbation analysis

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Abstract

We study a scalar-tensor cosmological model where the Einstein tensor is non-minimally coupled to the free scalar field dynamics. Using FRW metric, we investigate the behavior of scale factor for vacuum, matter and dark energy dominated eras. Especially, we focus on the inflationary behavior at early universe. Moreover, we study the perturbation analysis of this model in order to confront the inflation under consideration with the observational results.

1 Cosmology with non-minimal kinetic coupled gravity

Let us consider a free (without potential term) scalar field whose kinetic term is coupled both with the metric tensor $g_{\mu\nu}$ and Einstein tensor $G_{\mu\nu}$. We write the action as

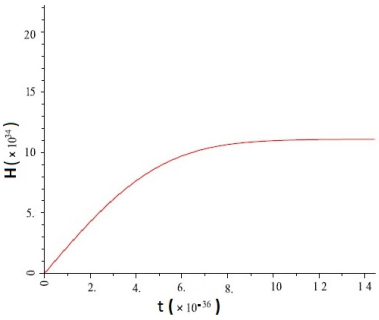
$$S=\int d^4x\sqrt{-g}\left[\frac{R}{8\pi}-(g^{\mu\nu}+\alpha G^{\mu\nu})\nabla_\mu\phi\nabla_\nu\phi-2\Lambda\right]+S_m, \tag{1.1}$$

where R is the Ricci scalar, α is a coupling parameter with dimension of $(length)^2$, Λ is a positive cosmological constant, and S_m is the matter action. Using the Friedmann-Robertson-Walker (FRW) flat ($k=0$) background metric the field equations are obtained

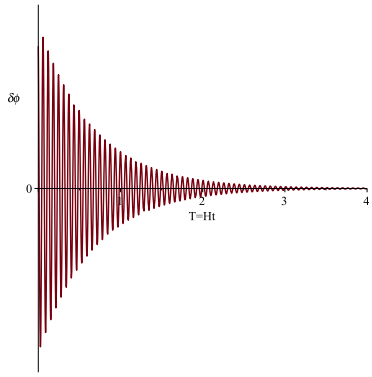
$$\begin{aligned}
3H^2 &= 4\pi\dot{\phi}^2\left(1-9\alpha H^2\right)+\Lambda+8\pi\rho_m,\\
2\dot{H}+3H^2 &= -4\pi\dot{\phi}^2\left[1+\alpha\left(2\dot{H}+3H^2+4H\ddot{\phi}\dot{\phi}^{-1}\right)\right]+\Lambda-8\pi p_m,\\
(\ddot{\phi}+3H\dot{\phi}) &- 3\alpha(H^2\ddot{\phi}+2H\dot{H}\dot{\phi}+3H^3\dot{\phi})=0.
\end{aligned} \tag{1.2}$$

Therefore, one obtains the total density and pressure respectively as $\rho_T=\rho_m+\frac{\Lambda}{8\pi}+\frac{1}{2}\dot{\phi}^2(1-9\alpha H^2)$ and $p_T=p_m-\frac{\Lambda}{8\pi}+\frac{1}{2}\dot{\phi}^2\left[1+\alpha\left(2\dot{H}+3H^2+4H\ddot{\phi}\dot{\phi}^{-1}\right)\right]$, where a dot denotes derivative with respect to t. Equation (1.2) can be easily integrated to $\dot{\phi}=\frac{\sqrt{2\lambda}}{a^3(1-3\alpha H^2)}$, where λ is a positive constant of integration. In the following, we will set up a cosmological model in a systematic way which includes an inflation era with suitable slow-roll conditions, an exit mechanism from inflation, a deceleration era, and an acceleration era of the universe.

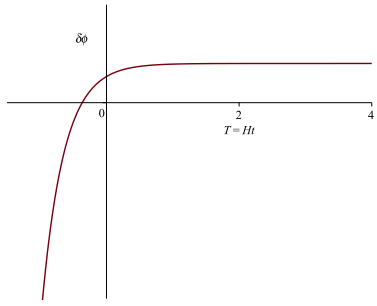
I) **Inflationary universe:** At very early universe we obtain $\{(1-9\alpha H^2)+\alpha\dot{H}(1+\frac{12\alpha H^2}{1-3\alpha H^2})\}=0$. One may solve this differential equation to obtain the plot $H(t)$ as depicted in Fig.1. As is seen in Fig.1, considering a typical small value of α, the behaviour for $H(t)$ is so favored to model an inflationary cosmology with an almost constant $\dot{H}\simeq 0$ and large H for $t>10^{-35}$ sec. Actually, the Hubble parameter H may suddenly (typically within 10^{-35} seconds) approach to the large and almost constant asymptotic value $H\equiv H_\alpha\simeq\sqrt{\frac{1}{9\alpha}}\simeq 1.2\times 10^{35}sec^{-2}$, provided that α is assumed to be a typically small parameter, $\alpha\sim 10^{-71}sec^2$. One may obtain the number of e-folding during the inflation as $N=\int_{t_i}^{t_f}Hdt=\int_{t_i}^{t_f}\sqrt{\frac{1}{9\alpha}}dt=\sqrt{\frac{1}{9\alpha}}(t_f-t_i)$. Within the typical short period of time $(t_i=10^{-35})<t<(t_f=10^{-33})$, required by particle physics, and



(a) The plot of $H(t)$ for a typical value of $\alpha = 10^{-71} sec^2$ showing the behaviour $H \simeq 1.2 \times 10^{35} sec^{-2}$ and $\dot{H} \simeq 0$ for $t > 10^{-35}$ sec.



(b) the behavior of short-wavelength perturbations with using the slow-roll approximation for $H =$ constant., $k \gg Ha$, $a_0 = 1$ and $T = Ht \simeq 4$.



(c) the behavior of long-wavelength perturbations with using the slow-roll approximation for $H =$ constant, $k \ll Ha$, $a_0 = 1$ and $2 < T = Ht < 4$.

using $\sqrt{\frac{1}{9\alpha}} \simeq 1.2 \times 10^{35}$, we obtain $N \sim 120$ well above $N \sim 60$ which is at least needed to overcome the problems of standard cosmology. If we assume that the initial size of the universe before inflation was about the Planck length $10^{-34}m$, then the 120 number of e-folding results in the final size of the universe at the end of inflation as large as $a(t_f) \sim 10^{22}m$. This large size will remove all the problems of standard cosmology.
II) **Radiation and Matter dominated universe:** After the inflationary era the scale factor a becomes exponentially large. At this time, namely t_f, the very fast decrease in the kinetic energy of the scalar field starts and can be balanced by the creation of baryoinc matter with the density and pressure related by the equation of state $p_m = \omega_m \rho_m$. At this stage, we have just two components left as follows $\rho_T = \rho_m + \frac{\Lambda}{8\pi}$, $p_T = p_m - \frac{\Lambda}{8\pi}$. Assuming an small cosmological constant in comparison with the sufficiently large values of matter density and pressure, the cosmological evolution of universe at this stage with ignorable cosmological constant is well known as follows [11, 8] I) for $\omega_m = \frac{1}{3}$ we have the radiation dominant era with the scaling behaviour $\rho_m \propto a^{-4}$ and time evolution $a(t) \propto t^{1/2}$, II) for $\omega_m = 0$ we have the matter dominant era with the scaling behaviour $\rho_m \propto a^{-3}$ and time evolution $a(t) \propto t^{2/3}$.
III) **Dark energy dominated universe:** At the late time and old universe, the scale factor becomes so large that $\rho_m \propto a^{-3} \ll \Lambda/8\pi$. This stage of evolution is governed by the cosmological constant $\rho_T = \frac{\Lambda}{8\pi}$, $p_T = -\frac{\Lambda}{8\pi}$, which represents the new phase of vacuum state as $p_T = -\rho_T = -\frac{\Lambda}{8\pi}$, where the cosmological constant plays the role of *Dark energy*. The cosmological evolution of this dark energy dominated universe is well known as de Sitter expansion [11, 8] $a(t) \propto \exp(H_\Lambda t)$, where $H_\Lambda = \sqrt{\frac{\Lambda}{3}}$.

2 Cosmic perturbation

we consider the field perturbation equation in the Newtonian Gauge as follows

I) **Inside the Hubble scale:** After a tedious but straightforward calculation we find the scalar field perturbation equation in terms of physical time in this form $(\ddot{\delta\phi}) + 3H(\dot{\delta\phi}) + \frac{k^2}{a^2}(\delta\phi) \approx 0$, where use has been made of the slow-roll approximation.(Figure 2)
II) **Evolution through Horizon Exit:** We obtain the equations for the perturbations in slow-roll regime result $(\ddot{\delta\phi}) + 3H(\dot{\delta\phi}) \approx 0$. This equation is also easily solved with H being constant.(Figure 3)

3 Vacuum fluctuation of the inflaton field

We can obtain the primordial power spectrum [10] $P_R(k) = \left[(\frac{H}{2\pi})(\frac{H}{\dot{\phi}})\right]^2_{k=aH}$ after a tedious but straightforward calculations, and the primordial tensor perturbations $P_T(k) = \frac{1}{\pi}\left[\frac{H}{2\pi}\right]^2_{k=aH}$. The tensor-to-scalar ratio

is then obtained as $r = \frac{P_T}{P_R} = \left[\frac{\dot{\phi}^2}{\pi H^2}\right]_{k=aH} = \left[\frac{9\phi^2}{\pi}\right]_{k=aH}$, Comparing with the upper bounds obtained by the recent Planck and BICEP2 measurements [13, 14, 15], we find $\varepsilon \simeq 2 \times 10^{-2}$ and

$$[\phi]_{k=aH} \simeq \begin{cases} 0.19\ M_P & Planck \\ 0.26\ M_P & BICEP2, \end{cases} \tag{3.1}$$

which predicts the vacuum expectation value of the scalar field, at the time of leaving the horizon, in terms of the Planck mass.

4 Conclusion and discussion

Motivated by the fact that the common inflationary scenarios usually need a scalar field potential to trigger the inflation, and that taking the proper inflaton potential without fine tuning and cosmological constant problems is still an unsolved issue, we have studied the cosmological implications of a kinetic coupled scalar-tensor gravity to establish a systematic inflation model which is capable of transition to the matter dominant and dark energy dominant eras. Moreover, we have studied the perturbation analysis of this inflation model in order to confront the inflation under discussion with the recent observational results.

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