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Cosmology of Lorentz violating Galileons

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Recently, mimetic dark matter (DM) theory which is a gravitational theory minimally coupled to a scalar field has been proposed, rapidly becomes a hot topic in the field. The scalar field has a property that its gradient is always time-like. We generalize the mimetic DM theory by adding higher derivative self-interactions of scalar field to the action. These interactions have the property that its equation of motion contain at most second time derivatives, and as a result do not produce Ostrogradski ghost instability. We will study the cosmological implications of the theory, and also analyze the role of non-minimal coupling between the scalar field and ordinary matter field in details.

1 The model

Recently, Chamseddin and Mukhanov proposed a method to construct a conformally invariant

metric [1]. Every theory written in terms of this new effective metric will becomes conformally invariant. To do this assume that ϕ

is a scalar field. The effective metric

$$\bar{g}_{\mu\nu} = (g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi)g_{\mu\nu}. \tag{1}$$

is then invariant under the transformation $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$ where Ω is an arbitrary function. One

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can easily prove that the scalar field must satisfy the condition $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi=-1$. This means that the gradient of ϕ should be time-like. The presence of such a time-like scalar filed breaks the Lorentz invariance of the theory. This conclusion is similar to that of Einstein-aether theory [2]. With the above comments, it is obvious that one can write the Einstein-Hilbert (EH) action constructed by $\bar{g}_{\mu\nu}$ as an EH action with an additional constraint on ϕ [3]

$$S=\int d^4x\sqrt{-\bar{g}(g,\phi)}R(\bar{g})=\int d^4x\sqrt{-g}[R(g)+\lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+1)]. \tag{2}$$

We generalize the action (2) by adding the most general self-interaction terms which produce at most second order time derivatives in the equation of motion [5]. These scalar fields are known as the "Galileons". The action then becomes

$$S=\int d^4x\sqrt{-g}[\kappa^2R+\alpha_3\mathcal{L}_3+\alpha_4\mathcal{L}_4+\alpha_5\mathcal{L}_5+\lambda(\phi_\mu\phi^\mu+1)+\mathcal{L}_m], \tag{3}$$

where $\mathcal{L}_m$ is the matter Lagrangian and $\phi_\mu\equiv\partial_\mu\phi$. We have also introduced the terms $\mathcal{L}_i$, $i=3,4,5$, defined as [4]

$$\mathcal{L}_3=(\phi_\alpha\phi^\alpha)\Box\phi, \tag{4}$$

$$\mathcal{L}_4=(\phi_\alpha\phi^\alpha)[2(\Box\phi)^2-2\phi_{\mu\nu}\phi^{\mu\nu}-\frac{1}{2}(\phi_\mu\phi^\mu)R], \tag{5}$$

$$\mathcal{L}_5=(\phi_\alpha\phi^\alpha)[(\Box\phi)^3-3(\phi_{\mu\nu}\phi^{\mu\nu})\Box\phi+2\phi_\mu{}^\nu\phi_\nu{}^\rho\phi_\rho{}^\mu-6\phi_\mu\phi^{\mu\nu}G_{\nu\rho}\phi^\rho]. \tag{6}$$

2 Cosmological implications

Consider a flat FRW metric with time-varying scalar field $\phi=\phi(t)$

$$ds^2=-dt^2+a^2(t)d\vec{x}^2. \tag{7}$$

We also assume that the universe is filled a perfect fluid with matter energy density $\rho(t)$, and pressure $p(t)$. The equations of motion can then be written as $\phi=t+c_1$ and

$$\frac{3}{2}(15\alpha_4 + 2\kappa^2)H^2 - 21\alpha_5 H^3 - 3\alpha_3 H - \rho + \lambda = 0, \tag{8}$$

$$(3\alpha_4 + 2\kappa^2 - 6\alpha_5 H)\dot{H} + \frac{3}{2}(3\alpha_4 + 2\kappa^2)H^2 - 6\alpha_5 H^3 + p = 0, \tag{9}$$

and

$$6(15\alpha_5 H^2 - 12\alpha_4 H + \alpha_3)\dot{H} + 90\alpha_5 H^4 - 108\alpha_4 H^3 + 18\alpha_3 H^2 - 2(3\lambda H + \dot{\lambda}) = 0, \tag{10}$$

where $H(t)$ is the Hubble parameter. The above equations can be easily solved for λ and ρ with the result

$$\lambda = 3H(\alpha_3 - 6\alpha_4 H + 5\alpha_5 H^2) + \frac{c_2}{a^3}, \tag{11}$$

$$\rho = \frac{3}{2}H^2(3\alpha_4 + 2\kappa^2 - 4\alpha_5 H) + \frac{c_2}{a^3}. \tag{12}$$

Assuming $p = \omega\rho$ the evolution equation of scalar factor takes the form

$$(2\kappa^2 + 3\alpha_4 - 6\alpha_5 H)\dot{H} + \frac{3}{2}(\omega + 1)\left(2\kappa^2 + 3\alpha_4 - 4\alpha_5 H\right)H^2 + \frac{\omega c_2}{a^3} = 0. \tag{13}$$

One can write the above equation in terms of the redshift z as

$$(1+z)h(z)[1+m-nh(z)]\frac{dh}{dz} = \frac{3}{2}(\omega+1)\left[1+m-\frac{2}{3}nh(z)\right]h^2(z) + \omega s(1+z)^3. \tag{14}$$

where $\alpha_4 = \frac{2\kappa^2}{3}m, \quad \alpha_5 = \frac{\kappa^2}{3}n_1, \quad c_2 = 2\kappa^2 s_1, \quad n_1 = \frac{n}{H_0}, \quad s = \frac{s_1}{H_0^2}.$

2.1 Dust dominated universe

Let us assume that the universe is filled with dust particles. This is the case for our universe if one does not assume dark matter and dark energy components for the universe. In this paper we want to explain the effects of DE by a scalar field ϕ. The effects of this scalar field as a DM has been considered extensively in the literature. In figure (1) we have depicted the evolution of scale factor and the deceleration parameter in terms of the redshift z. As one can see from the figure, the deceleration parameter is negative resulting in an accelerated expanding universe. The Hubble parameter is a decreasing function of time with non-zero derivative. This means that the accelerated expansion of the universe is not as strong as exponential.

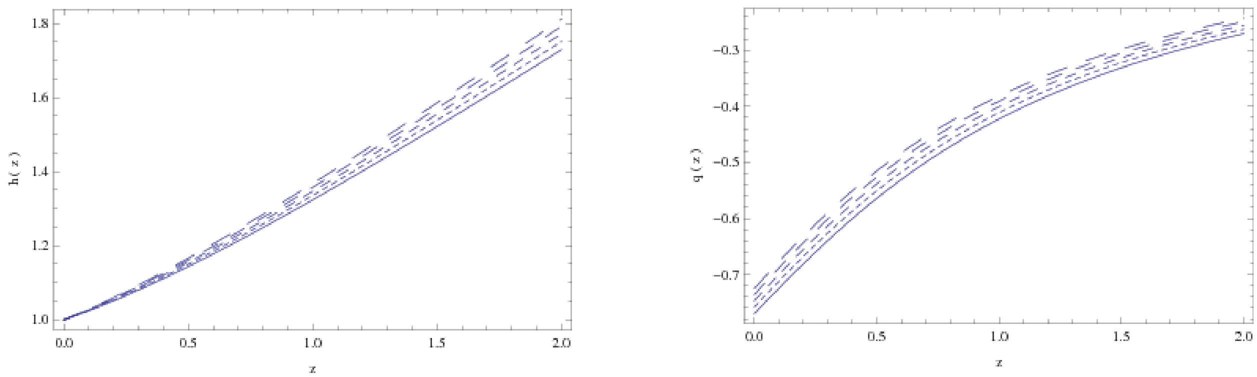

Figure 1: Variation as a function of z of the Hubble parameter $h(z)$ (left figure) and of the deceleration parameter $q(z)$ (right figure) for a dust universe, for $m = 0.002$, $s = 0.20$, and for different values of the parameters n: $n = 1.653$ (solid curve), $n = 1.663$ (dotted curve), $n = 1.673$ (short dashed curve), $n = 1.683$ (dashed curve), and $n = 1.693$, respectively.

2.2 Role of the coupling ϕT

The coupling ϕT between the scalar field and the trace of energy-momentum tensor will change the evolution equation of scalar field for dust dominated universe to

$$\frac{dh(\tau)}{d\tau} = \frac{-35\Delta\tau\Lambda(\tau) + 35\Delta\eta\tau h(\tau) + 5\theta(9\Delta\tau + 4)h^3(\tau) - 21h^2(\tau)\left[\Delta(3\sigma+5)\tau + \sigma + 5\right]}{(\Delta\tau + 2)\left[7(\sigma+5) - 10\theta h(\tau)\right]}, \quad (15)$$

where we have written the equation in terms of time. We have also defined

$$\alpha_3 = \frac{2\kappa^2}{3}H_0\eta, \alpha_4 = \frac{2\kappa^2}{15}\sigma, \alpha_5 = \frac{2\kappa^2}{21H_0}\theta, H = H_0 h,$$

$$\rho = 2\kappa^2 H_0^2 r,\ p = 2\kappa^2 H_0^2 P,\ \lambda = 2\kappa^2 H_0^2 \Lambda,\ \alpha_6 = H_0\Delta,\ t = \frac{\tau}{H_0}, \quad (16)$$

In this case the evolution of Hubble and deceleration parameters can be obtained numerically which we have shown in figure (2). One can see from the figures that as time passes, the Hubble parameter decreases rapidly, resulting in a decelerating universe. As t approaches to H_0^{-1}, the universe enters to an accelerating phase which can be seen from the q line. At the late time the deceleration parameter approaches unity and the Hubble parameter becomes zero signaling an exponentially expanding universe. As a result, adding the non-minimal coupling between the scalar field and the trace of energy-momentum tensor will improve the dynamical behavior of the model.

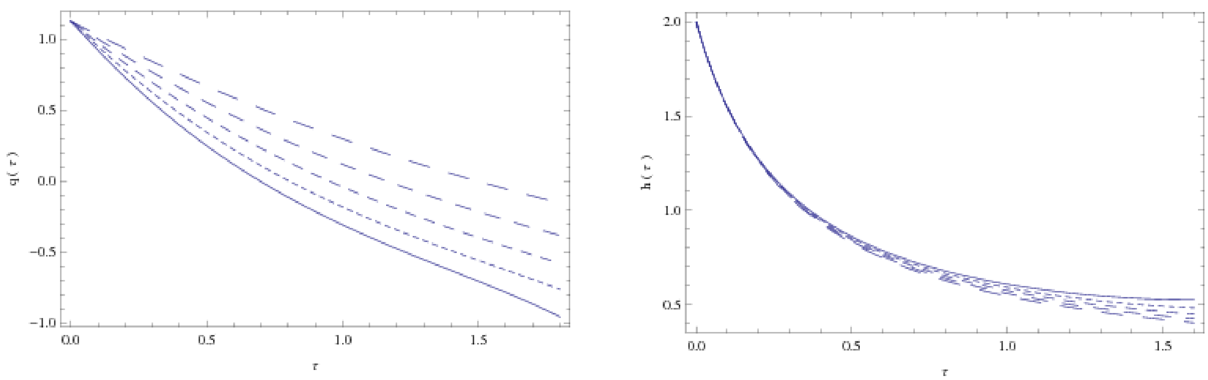

Figure 2: Variation with respect to the dimensionless time τ of the Hubble parameter $h(\tau)$ (left figure) and of the deceleration parameter $q(\tau)$ (right figure), for a dust universe, for different values of the parameters Δ: $\Delta = -0.75$ (solid curve), $\Delta = -0.65$, (dotted curve), $\Delta = -0.55$

(short dashed curve), $\Delta = -0.45$ (dashed curve), and $\Delta = -0.35$ (long dashed curve), respectively.

3 Conslusions

A gravitational theory coupled to a constraint scalar filed is considered. It has been proved that the scalar field can behave as a cold dark matter sector of the universe [1]. In this paper we have shown that this theory can also explain the accelerated expansion of the universe as well, specially in the case of dust dominated universe. We have also shown that the non-minimal coupling between the scalar field and the baryonic matter can approve the dynamics of the system, results in a decelerating universe in the early times, entering to an accelerated expanding state at late times [5].

References

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