Two-loop snail diagrams: relating neutrino masses to dark matter

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invisibles
neutrinos, dark matter & dark energy physics

- Neutrino mass<< Electron mass

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Who cares?!

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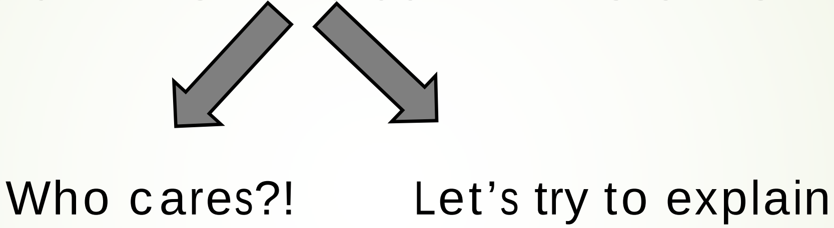


Who cares?!

Let’s try to explain

new mass scale

- Seesaw type I

$$\begin{bmatrix} 0 & m_D \\ m_D & m_M \end{bmatrix}$$

$$m_D \ll m_M$$

Loop suppression

- Zee model
- Babu model

- Symmetries to forbid lower order contribution

- Dark matter

Some examples

- SLIM model

1) C. Boehm, Y. F., T. Hambye, S. Palomares-Ruiz1 and S. Pascoli, PRD 77 (2008) 43516;

2) Y.F., PRD 80 (2009)

3) Y.F. and M. Hashemi, JHEP 2008

4) Y.F., Mod Phys Lett A25 (2010)

- AMEND
- 5) Y.F., S. Pascoli and M. Schmidt, JHEP 2010
- Scotogenic model

Y.F. AND E. MA, PRD (2012)

SLIM model

- New fields:
- Majorana Right-handed neutrino
- SLIM=Scalar as LIght as MeV
- Effective Lagrangian:

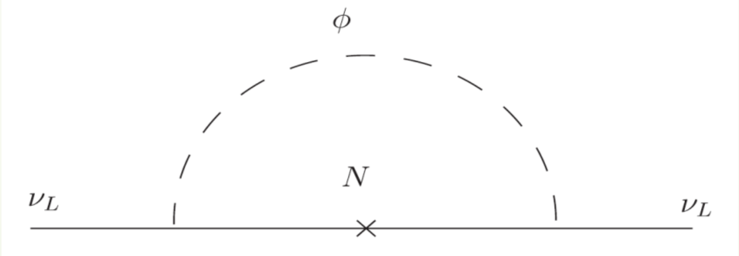
$$\mathcal{L}_I \supset g\phi\bar{N}\nu$$

- New parameters:

$$g \quad m_\phi \quad m_N$$

neutrino masses

- In this scenario, SLIM does not develop any VEV so the tree level neutrino mass is zero.
- Radiative mass in case of real scalar:



Ultraviolet cutoff Λ

$$m_\nu = \frac{g^2}{16\pi^2} m_N \left[\ln\left(\frac{\Lambda^2}{m_N^2}\right) - \frac{m_\phi^2}{m_N^2 - m_\phi^2} \ln\left(\frac{m_N^2}{m_\phi^2}\right) \right]$$

SLIM as a real field

Z_2

- symmetry: $\phi \longrightarrow -\phi\ , \qquad N \longrightarrow -N$

$$\xcancel{\bar{N}\, L \cdot H}$$

Annihilation cross-section

$$\langle \sigma(\phi\phi \to \nu\nu)\mathrm{v}_r\rangle = \langle \sigma(\phi\phi \to \bar{\nu}\,\bar{\nu})\mathrm{v}_r\rangle$$

$$\simeq \frac{g^4}{4\pi}\frac{m_N^2}{(m_\phi^2 + m_N^2)^2},$$

$$g \simeq 10^{-3}\sqrt{\frac{m_N}{10\ \mathrm{MeV}}}\left(\frac{\langle \sigma \nu_r \rangle}{10^{-26}\ \mathrm{cm}^3/\ \mathrm{s}}\right)^{1/4}\left(1 + \frac{m_\phi^2}{m_N^2}\right)^{1/2}$$

Linking dark matter and neutrino mass

$$\langle \sigma v_r \rangle \sim 10^{-26}\ \mathrm{cm}^3/\mathrm{s}.$$

$$\Lambda \sim \mathrm{E}_{\mathrm{electroweak}} \sim 200\ \mathrm{GeV}$$

$$0.05\ \mathrm{eV} < m_\nu < 1\ \mathrm{eV},$$

$$O(1)\ \mathrm{MeV} \lesssim m_N \lesssim 10\ \mathrm{MeV}.$$

Bounds on SLIM mass

$$m_\phi < M_N$$

$$O(1)\ \mathrm{MeV} \lesssim m_N \lesssim 10\ \mathrm{MeV}.$$

A way to test the scenario

$$g \simeq 10^{-3}\sqrt{\frac{m_N}{10\ \mathrm{MeV}}}\left(\frac{\langle \sigma \nu_r \rangle}{10^{-26}\ \mathrm{cm}^3/\,\mathrm{s}}\right)^{1/4}\left(1+\frac{m_\phi^2}{m_N^2}\right)^{1/2}$$

$$3 \times 10^{-4} \lesssim g \lesssim 10^{-3}$$

An economic model embedding *real* SLIM

- YF, “*Minimal model linking two great mysteries: Neutrino mass and dark matter*”, PRD 2009

1) An electroweak singlet scalar, η

2) Two (or more) Majorana right-handed neutrinos N_i

$$\mathcal{L} = -g_{i\alpha}\bar{N}_i\Phi^\dagger \cdot L_\alpha - \frac{M_i}{2}\bar{N}_i^c N_i \ ,$$

3) A scalar electroweak doublet

$$\Phi^T = [\phi^0 \ \phi^-]$$
$$\phi^0 \equiv (\phi_1 + i\phi_2)/\sqrt{2}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \eta \\ \phi_1 \end{bmatrix}$$

Light and heavy

- Light sector: Dark matter candidate δ_1 and N_1

(Supernova, meson decay and ...)

Heavy sector: δ_2 ϕ_2 ϕ^-

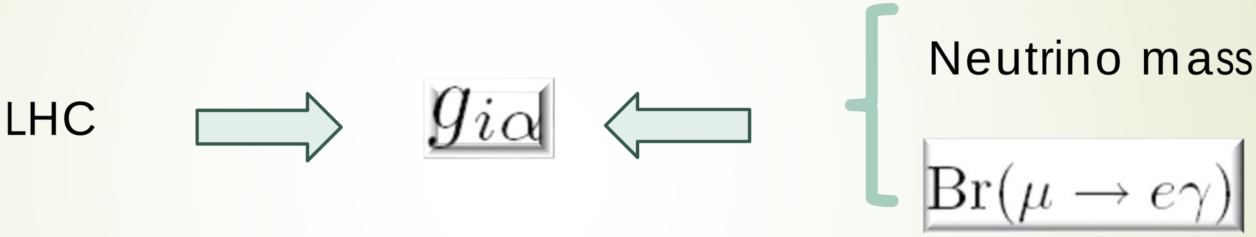
⇩

Lepton Flavor Violating rare decays, $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$

Magnetic dipole moment of the muon

Production at LHC

Interesting aspect



LHC

$g_{i\alpha}$

Neutrino mass

$\mathrm{Br}(\mu \to e\gamma)$

- Recipes and Ingredients for Neutrino Mass at Loop Level

Y.F., S. Pascoli and T. Schmidt, JHEP 1303 (2013) 107

Weinberg operator

effective dimension 5 operator, $(HL)(HL)$

$$(H^\dagger H)^m (HL)(HL)$$

Weinberg operator

effective dimension 5 operator, $(HL)(HL)$

$$\mathcal{O}_5 \sim \left(L^T\, C\, i\tau_2\, H\right)\left(H^T\, i\tau_2\, L\right)$$

$$(H^\dagger H)^m (HL)(HL)$$

General n-loop contribution

$$m_\nu \sim \left(\frac{g^2}{16\pi^2}\right)^n \left(\frac{\langle H\rangle^2}{m_{\text{New}}}\right) \left[1, \left(\log\frac{\Lambda}{m_{\text{New}}}\right)^n\right]$$

Λ is the ultraviolet (UV) cut-off scale of the model satisfying $\Lambda \gg m_{\text{New}}$.

$m_{\text{New}} \sim 1$ TeV, $m_\nu \sim 0.1 - 1$ eV

$\Lambda/m_{\text{New}} \sim 10$ and $n = 2$,

$$\Downarrow$$

$$g \sim 10^{-3}.$$

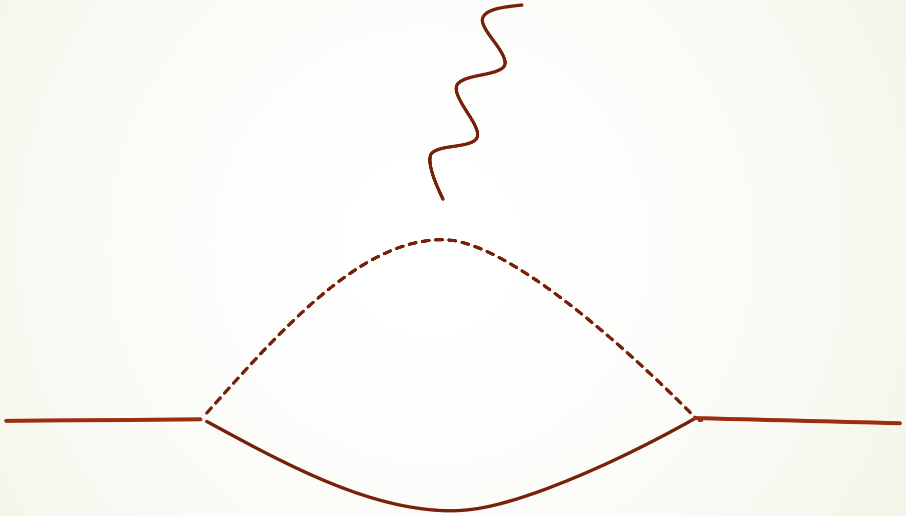
General n-loop contribution

$$m_\nu \sim \left(\frac{g^2}{16\pi^2}\right)^n \left(\frac{\langle H\rangle^2}{m_{\text{New}}}\right)\left[1, \left(\log\frac{\Lambda}{m_{\text{New}}}\right)^n\right]$$

n ↑ g ↑

$n = 3$ ⇒ $g \sim 0.01 - 0.1$

$$l_\alpha \to l_\beta \gamma$$



$$\Gamma(l_\alpha \to l_\beta \gamma) \sim \frac{g_\alpha^2 g_\beta^2}{16\pi(16\pi^2)^2} \frac{m_\alpha^5}{Max[m_S^4, m_F^4]}$$

$$Br(\mu \to e\gamma) < 5.7 \times 10^{-13} \ ,$$

$$Br(\tau \to e\gamma) < 3.3 \times 10^{-8}$$

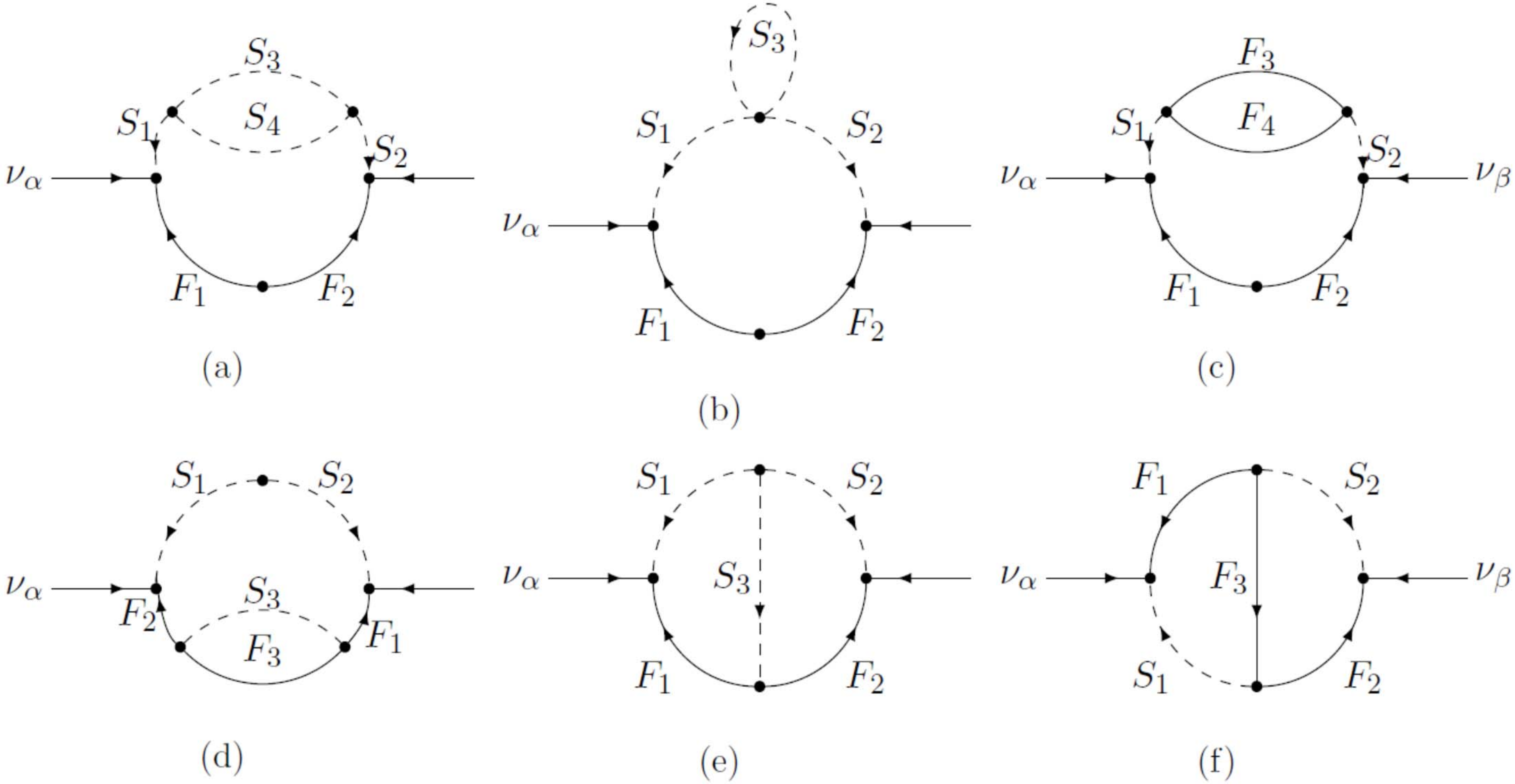
$$Br(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$$

$$g_e g_\mu \lesssim 10^{-3} \frac{\mathrm{Max}(m_S^2, m_{F_1^-}^2)}{\mathrm{TeV}^2}$$

$$g_e g_\tau, g_\mu g_\tau \lesssim \frac{\mathrm{Max}(m_S^2, m_{F_1^-}^2)}{\mathrm{TeV}^2}.$$

- Two-loop might be preferred.
- See however, Ahriche, McDonald and Nasri, 1505.04320 which advocates three-loop.

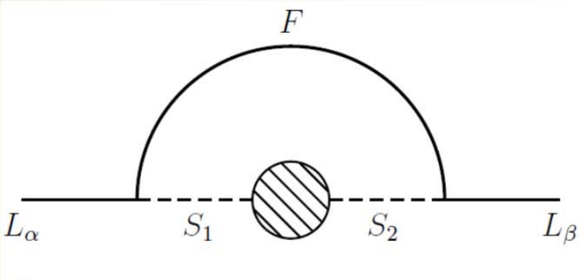
Two loop diagrams



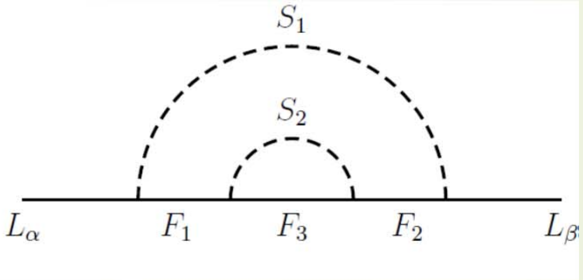
(a) (b) (c)

(d) (e) (f)

Two loop diagrams

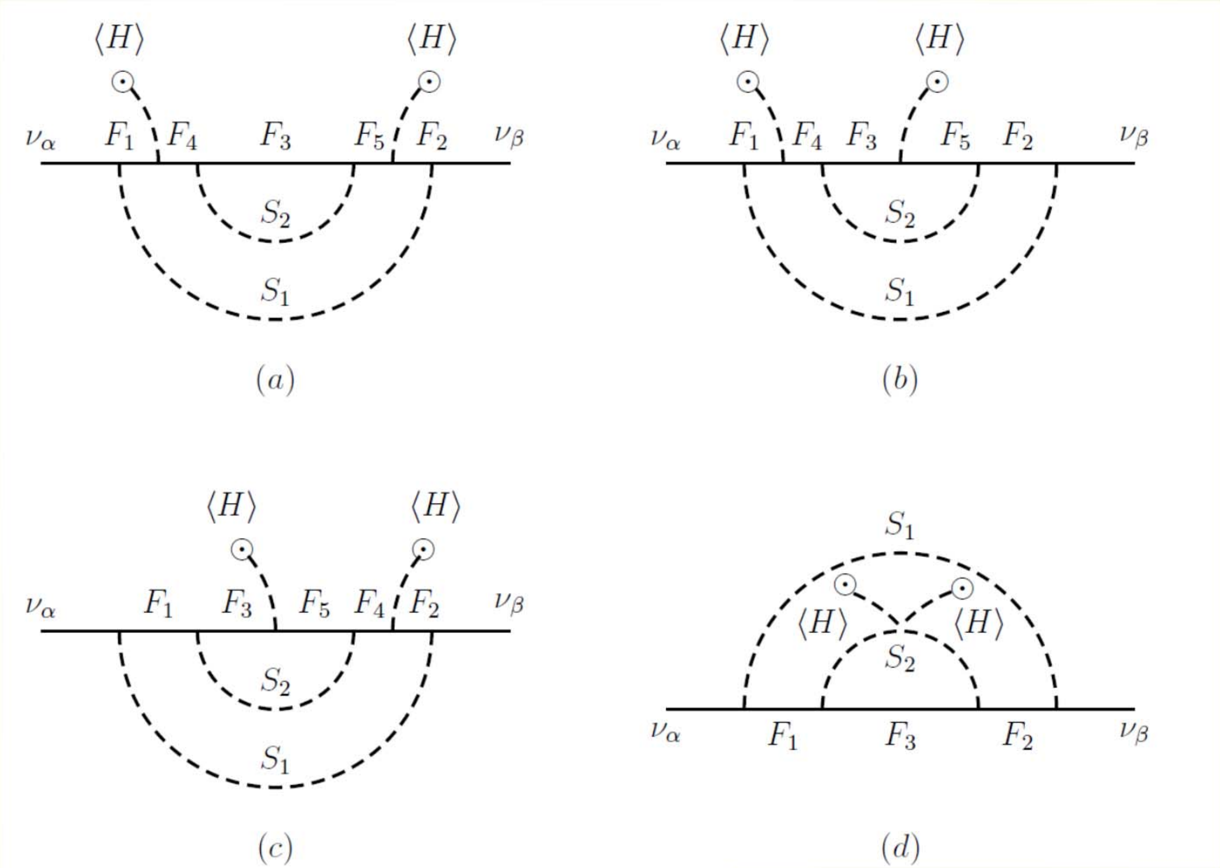


$SSHH$



$FFHH$

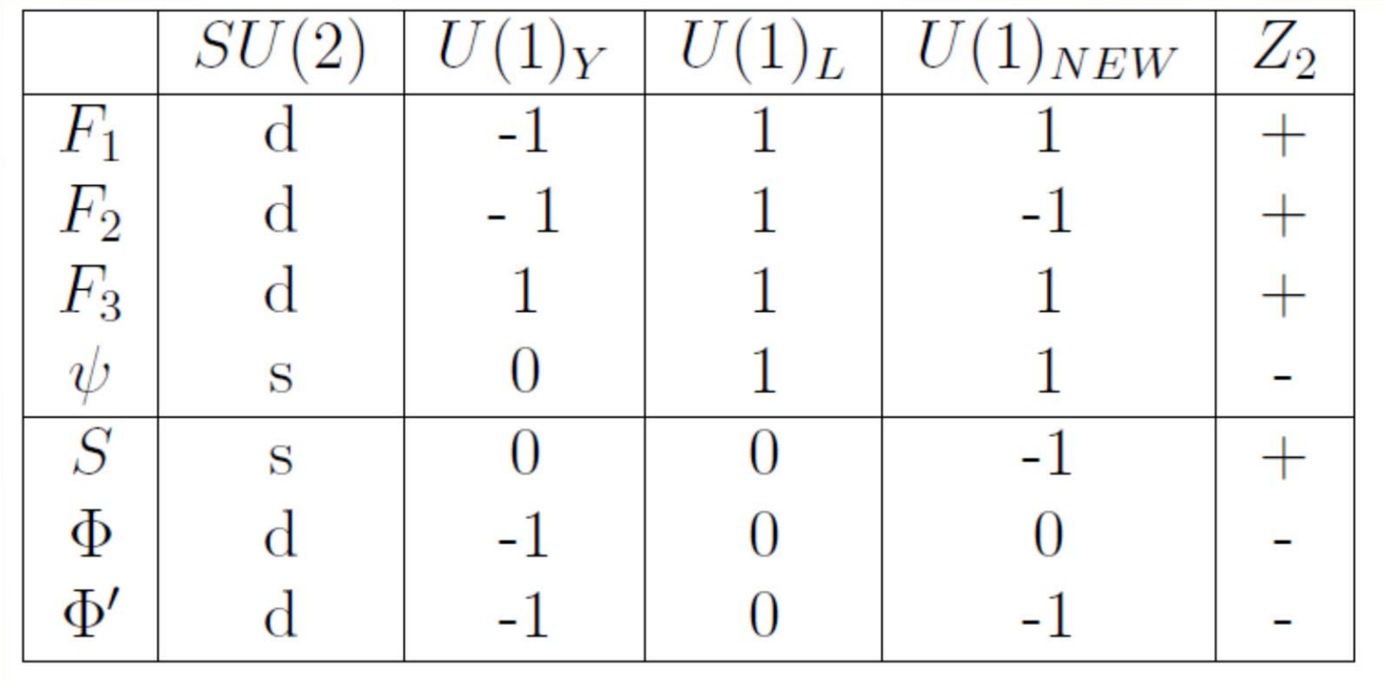
Two-loop snail diagrams: relating neutrino masses to dark matter



$F_4\,F_5,\;\; F_4\,F_2\,H$ and $F_1\,F_4\,H$.

- Two-loop snail diagrams: relating neutrino masses to dark matter

JHEP 05 (2015) 029



	$SU(2)$	$U(1)_Y$	$U(1)_L$	$U(1)_{NEW}$	Z_2
F_1	d	-1	1	1	+
F_2	d	- 1	1	-1	+
F_3	d	1	1	1	+
ψ	s	0	1	1	-
S	s	0	0	-1	+
Φ	d	-1	0	0	-
Φ'	d	-1	0	-1	-

$$m_M (F^a_{2R})^T c F^b_{3R} \epsilon_{ab} + m'_M (F^a_{2L})^T c F^b_{3L} \epsilon_{ab} + \text{H.c.}$$

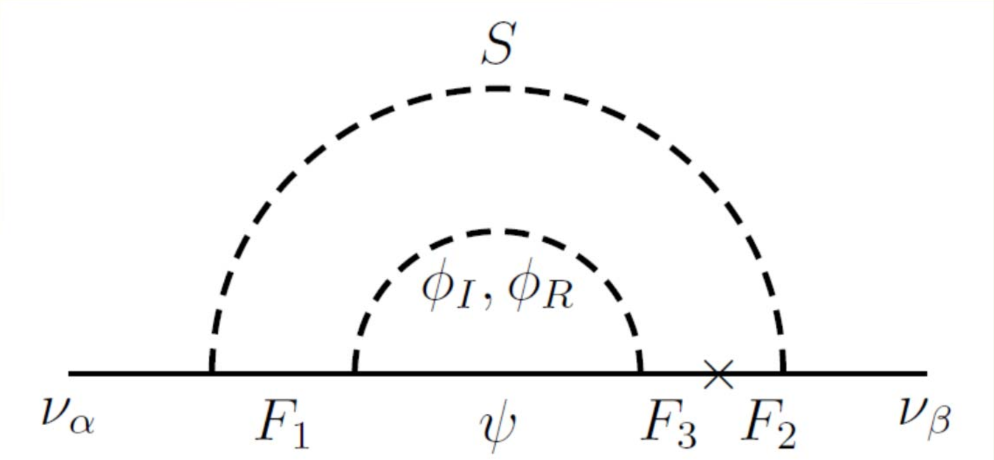
$$\mathcal{L}_{Yukawa} = g_\alpha S^\dagger F_{1R}^\dagger L_\alpha + h_\alpha S F_{2R}^\dagger L_\alpha + Y_{R\alpha}\Phi'^\dagger \psi_R^\dagger L_\alpha +$$

$$Y_1\Phi^\dagger\psi_L^\dagger F_{1R} + Y_2\epsilon_{ab}\Phi^a\psi_L^\dagger F_{3R}^b + Y_1'\Phi^\dagger\psi_R^\dagger F_{1L} + Y_2'\epsilon_{ab}\Phi^a\psi_R^\dagger F_{3L}^b + \mathrm{H.c.}$$

$$(\lambda(H^a\Phi^b\epsilon_{ab})^2 + \mathrm{H.c}) \quad \text{and} \quad \lambda'|H^\dagger\Phi|^2.$$

$$\Phi^0 \equiv (\phi_R + i\phi_I)/\sqrt{2}..$$

$$m_R^2 - m_I^2 = \lambda\langle H^0\rangle^2.$$



$$m_\nu \sim (0.01 - 0.1 \text{ eV}) Y_1 Y_2 \frac{g \times h}{10^{-1} \times 10^{-2}} \frac{m_M}{5 \text{ GeV}} \frac{(m_R^2 - m_I^2)/m_{max}^2}{1/20}.$$

Dark matter candidate

$$\begin{tikzcd} & U(1)_{NEW} \times Z_2 \arrow[dl] \arrow[dr] & \\ \phi_I & & \psi \end{tikzcd}$$

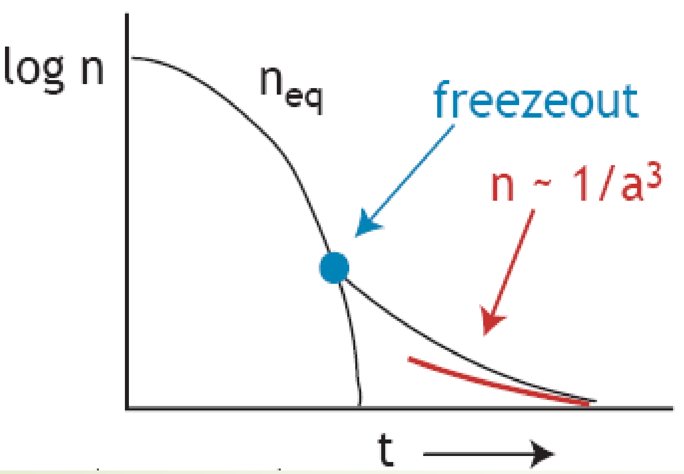
Thermal freeze-out

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle \left(n^2 - n_{eq}^2\right)$$

$$H \sim T^2/m_{\rm Pl}$$

$$T \ll m$$

$$n_{eq} = \frac{g}{(2\pi)^{3/2}}(mT)^{3/2}e^{-m/T}$$



$$e^{-m/T_F} \sim \frac{3\sqrt{T_F/m}(2\pi)^{3/2}}{M_{Pl}m\langle \sigma v \rangle g}$$

Dependence of m/T_f on mass is very weak. Varying

Mass from O(MeV) to O(100 GeV) (by 5 orders of magnitude), varies only between 10 to 25!

Dependence on parameters

$$\Omega_{DM} = \frac{nm}{\rho_c} \propto \frac{m/T_f}{\langle \sigma v \rangle}$$

m/T_f has a value between 10 to 30. So, the DM density is practically independent of the mass of the DM candidate and is solely determined by its annihilation cross-section.

$$\langle \sigma_{tot} v \rangle = 3 \times 10^{-26}\ \mathrm{cm}^3 \mathrm{sec}^{-1}$$

Coannihilation

$$\phi_I \phi_R \to Z^* \to SM$$

$(m_R - m_I)/m_R$ should be smaller than ~ 0.05

$$m_R^2 - m_I^2 = \lambda \langle H^0 \rangle^2$$

$$\langle \sigma(\phi_I + \phi_R \to Z^* \to f + \bar{f})v \rangle = \frac{16}{3\pi} N_C G_F^2 \frac{(a_L^2 + a_R^2)(m_I v)^2}{(1 - 4m_I^2/m_Z^2)^2}$$

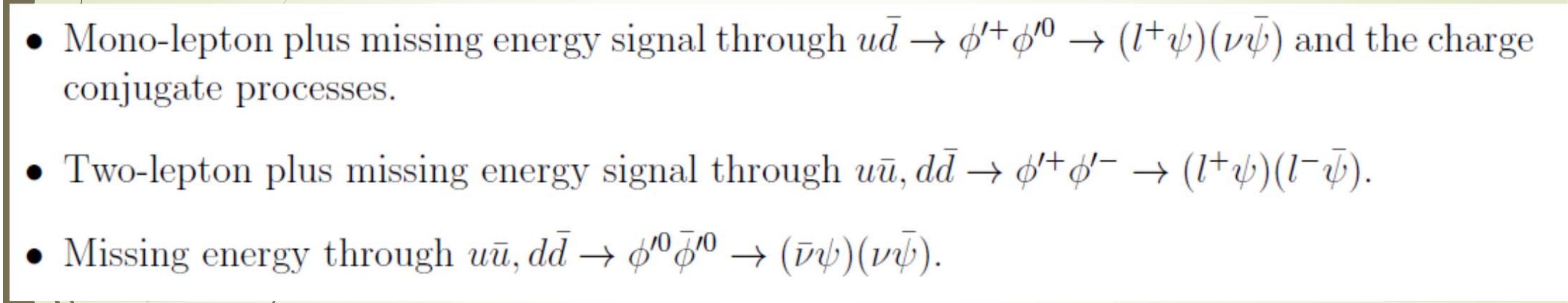
Main dark matter component

$$\langle \sigma(\psi\bar{\psi} \to \ell_\alpha \bar{\ell}_\alpha) v \rangle = \frac{|Y_{R\alpha}|^4}{32\pi} \frac{m_\psi^2}{(m_\psi^2 + (m_{\phi'^-})^2)^2}.$$

$$\langle \sigma_{tot} v \rangle = 3 \times 10^{-26}\ \mathrm{cm}^3\mathrm{sec}^{-1}$$

$$m_{\phi'^-},\ m_{\phi'^0} \leq 1.4 Y_{R\alpha}^2 \mathrm{TeV}$$

LHC signals



- Mono-lepton plus missing energy signal through $u\bar{d} \to \phi'^+\phi'^0 \to (l^+\psi)(\nu\bar{\psi})$ and the charge conjugate processes.
- Two-lepton plus missing energy signal through $u\bar{u}, d\bar{d} \to \phi'^+\phi'^- \to (l^+\psi)(l^-\bar{\psi})$.
- Missing energy through $u\bar{u}, d\bar{d} \to \phi'^0\bar{\phi}'^0 \to (\bar{\nu}\psi)(\nu\bar{\psi})$.

G. Aad *et al.* [ATLAS Collaboration], JHEP **1405**, 071 (2014)

G. Aad *et al.* [ATLAS Collaboration], JHEP **1410**, 96 (2014)

Muon and electron: >325 GeV
Tau: >90 GeV

ILC signals

$$g_\alpha S^\dagger \bar{F}_{1R} L_\alpha + h_\alpha S \bar{F}_{2R} L_\alpha$$

$$e^- e^+ \to S\bar{S}..$$

$$\Gamma(S \to l_\alpha^- F_1^+) \propto g_\alpha^2 \text{ and } \Gamma(S \to l_\alpha^+ F_{2,3}^-) \propto h_\alpha^2$$

F_i^+ can decay into $\phi^+\psi$ and $\phi^+ \to (W^+)^*\phi_I \to \nu l^+ \phi_I, q\bar{q}\phi_I$..

$$\Gamma(S \to l_\alpha^- F_1^+) \propto g_\alpha^2 \text{ and } \Gamma(S \to l_\alpha^+ F_{2,3}^-) \propto h_\alpha^2.$$

$$\begin{aligned}
e^+e^- &\to l_\alpha^+ + l_\beta^- + l_\gamma^+ + l_\theta^- + \text{missing energy} \ ; \\
e^+e^- &\to l_\alpha^+ + l_\beta^- + l_\gamma^+ + 2 \text{ jets} + +\text{missing energy} \ ; \\
e^+e^- &\to l_\alpha^+ + l_\beta^- + l_\gamma^- + 2 \text{ jets} + \text{missing energy} \ ; \\
e^+e^- &\to l_\alpha^+ + l_\beta^- + 4 \text{ jets} + \text{missing energy} \ ,
\end{aligned}$$

$h_\alpha^2 h_\beta^2.$

$g_\alpha^2 g_\beta^2$

$$\Gamma(S \to l_\alpha^- F_1^+) \propto g_\alpha^2 \quad \Gamma(\bar{S} \to l_\beta^- F_{2,3}^+) \propto h_\beta^2$$

$$e^+e^- \to l_\alpha^- + l_\beta^- + 4 \text{ jets} + \text{missing energy}$$

Summary

- We presented a model that contribute to neutrino mass via “two-loop snail diagram.”
- Phenomenological consequences are rich.